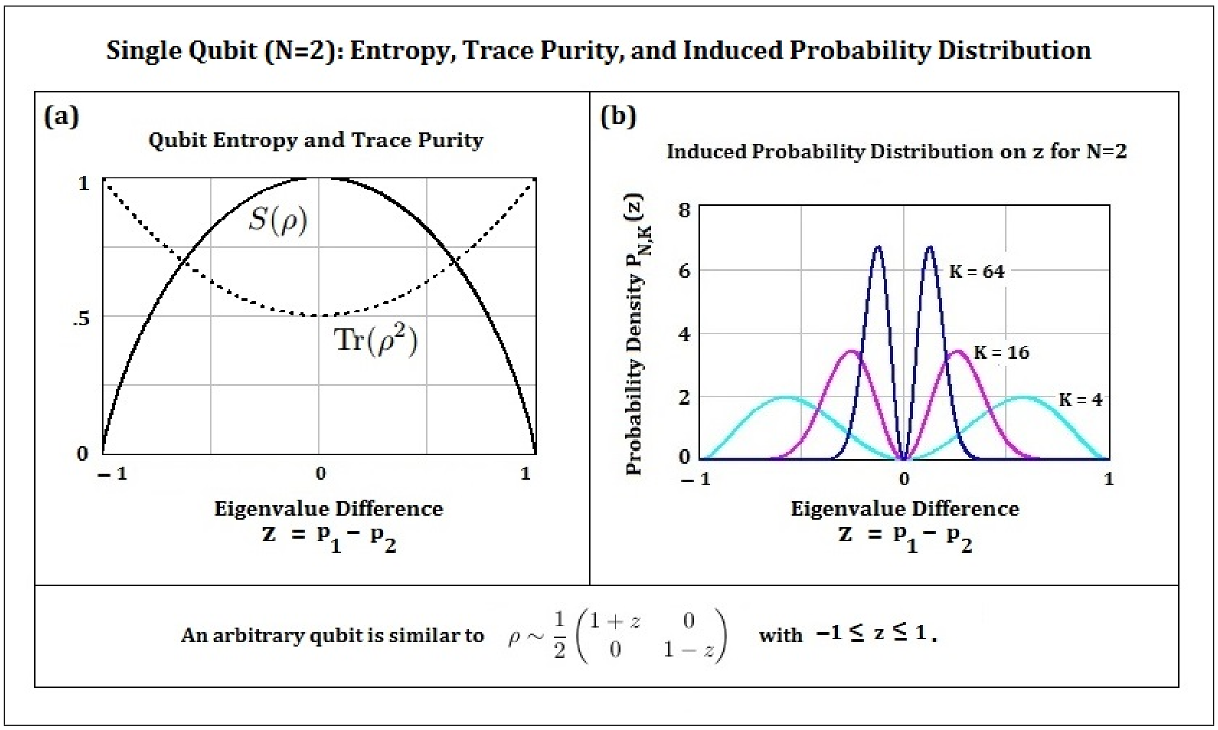

Figure 1: Qubit purity and induced probability distributions. (a) Von Neumann entropy and trace purity for a qubit with bloch vector length $|z|$. (b)Induced probability distribution on the eigenvalue simplex for $N = 2$. The probability distribution $P_{N,K}(z)$ is induced assuming that all pure states are equally likely in the enveloping NK dimensional pure Hilbert space. The qubit reduced density matrix has eigenvalues $p_{\pm} = \frac{1 \pm z}{2}$. This probability distribution is to be integrated over the eigenvalues, not over z, so in this form it appears to be normalized to 2. For large K, it is overwhelmingly likely that a random qubit will be found near the maximally mixed state.

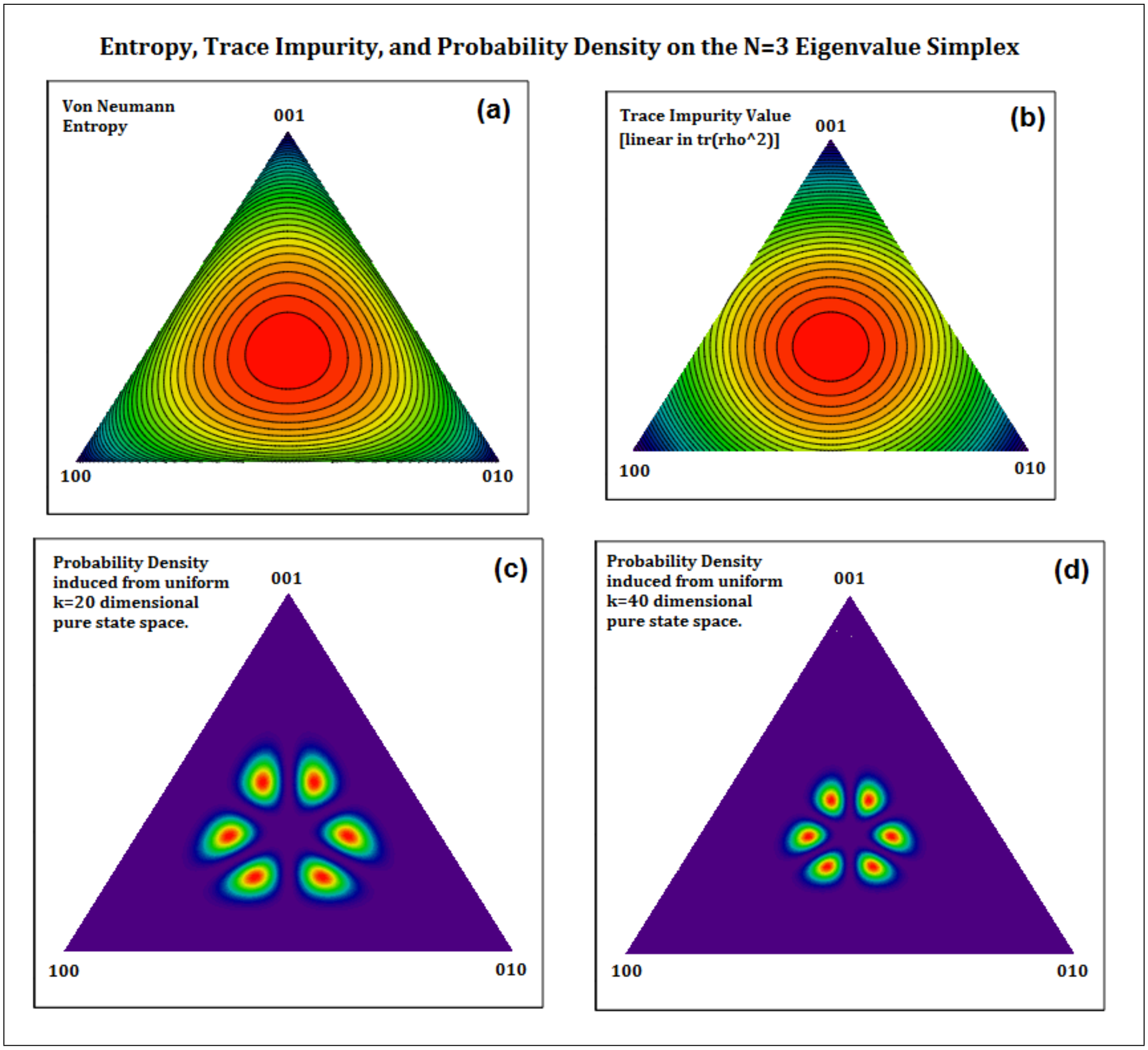

Figure 2: Entropy, trace impurity, and probability density on the $N = 3$ eigenvalue simplex. The $N - 1$ dimensional simplex of allowed eigenvalues is the intersection of the cube $0 \leq p_i \leq 1$ with the plane $\sum_i p_i = 1$. The three corners represent the pure states, the edges represent mixtures of two states, and the bulk contains mixtures of all three states. The center of the triangle represents the maximally mixed state. The corner labels represent the coordinates (p_1, p_2, p_3) and allow the simplex to be embedded in unrestricted eigenvalue space. All panels show colored contour plots, with red being a large value and purple being a small value. The panels show: (a) Von Neumann entropy $S = -\text{Tr}(\rho \log \rho)$; (b) Trace Impurity, which is linear in $\text{Tr}(\rho^2)$ and takes values from 0 (pure) to 1 (maximally mixed); (c) the probability distribution $P_{N,K}$ with $K = 20$; (d) the probability distribution $P_{N,K}$ with $K = 40$. Trace impurity contours lie on circles centered at the maximally mixed state. The probability distributions are induced from a uniform measure on pure states in the enveloping NK dimensional Hilbert space. For large K, it becomes overwhelmingly likely that a random state will be near the maximally mixed state.

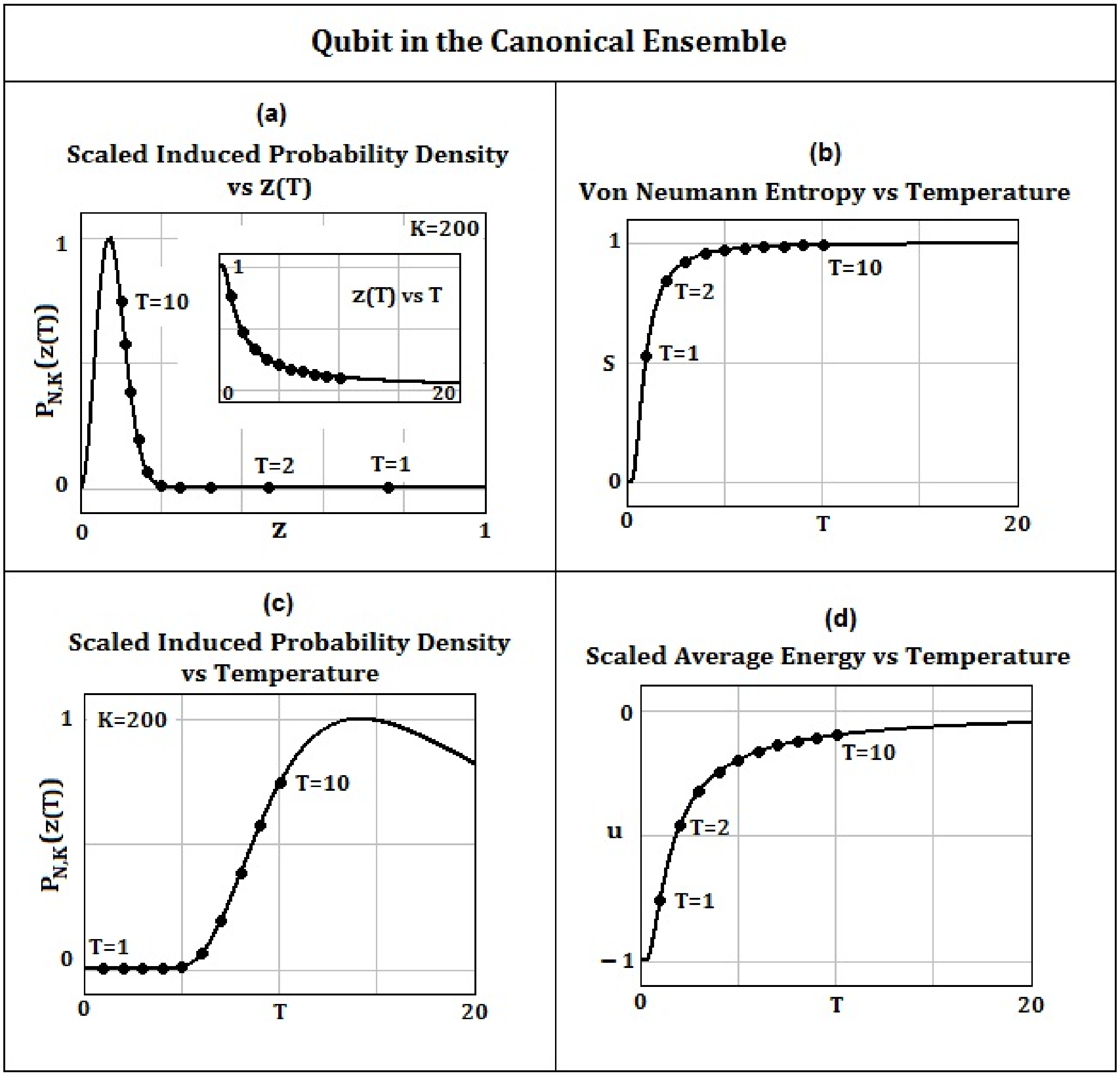

Figure 3: Qubit in the canonical ensemble. A qubit is assumed to have energy eigenvalues $E_{\mp} = E_0 \mp \frac{\Delta E}{2}$ so that the the energy difference between the two energy eigenstates is ΔE. In the figure we take $E_0 = 0$. The unitless temperature $T = \frac{\beta^{-1}}{\Delta E/2}$ is scaled by half the energy difference, where β is the lagrange multiplier appearing in the constrained maximization of the Von Neumann entropy, and may be identified with $\beta^{-1} = k_B T_K$. Panels (a,c) show the probability density $P_{N,K}$ (with $N = 2$ and $K = 200$) induced from the uniform distribution on pure states in the enveloping NK dimensional Hilbert space, after rescaling the distribution by its maximum value. The inset in (a) shows the bloch vector length $z(T)$. Panels (b,d) respectively show Von Neumann entropy and ensemble-averaged energy vs temperature. The unitless energy is $u = \frac{U}{\Delta E/2}$. The induced probability distribution implies that nearly maximally mixed states are overwhelmingly likely for large K. However, it is clear from the figure that for a range of temperatures which is not small compared to the energy scale, the sytem takes an unlikely low-entropy configuration in the canonical ensemble. Therefore, this uniformly-induced probability distribution cannot be an adequate way to describe thermalization into the canonical ensemble.