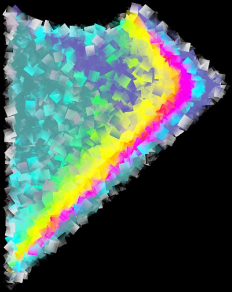
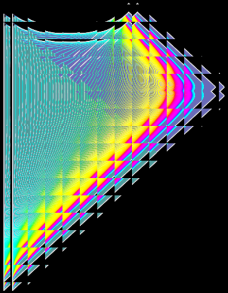
Harvard BHI – Apr 2026



Penrose Diagrams and Unitarity in Black Hole Evaporation

Joseph Schindler, Universitat Autònoma de Barcelona




JS


Evan Frangipane


Amita Kuttner

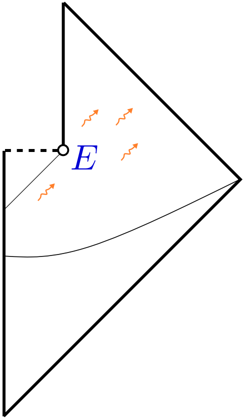

Anthony Aguirre

(Algorithms, 2018) arxiv:1802.02263
(Evaporation, 2019) arxiv:1907.04879
(Unitarity, 2021) arxiv:2012.07973

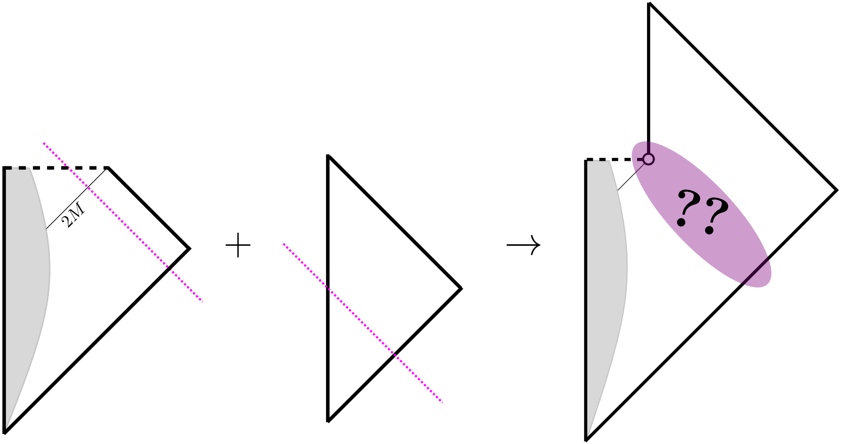
Grup d'Informació Quàntica

$S \leftarrow A/4$ since 2022.

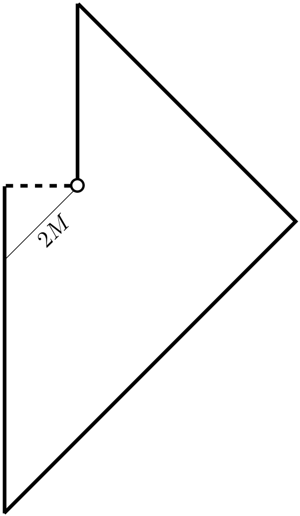
- The Penrose diagram for BH evaporation.
- Originally introduced by Hawking in 1975 *Particle Creation by Black Holes*.
- Versions of it are central to many discussions of BH evaporation, unitarity, and information loss.



- Yet, its precise physical content is ambiguous and unclear, many inequivalent versions in the literature.
- Metric? Coordinates? Physical model?
- Usually drawn based on hand-sketched assumptions $\longrightarrow$ not based on a mathematical derivation.


2M
+
→
??

Where is $r = 2M$?



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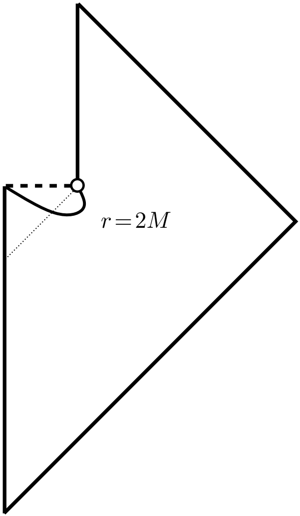
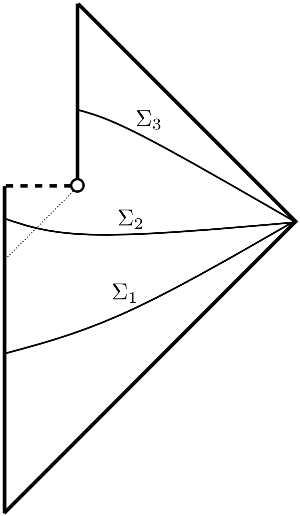
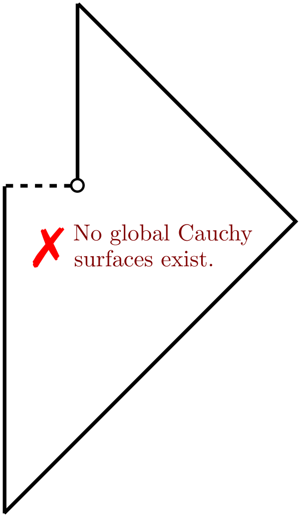


Where are Cauchy surfaces?



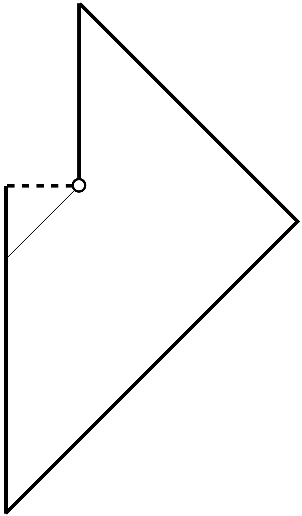
Where are Cauchy surfaces?



Other ambiguities like:

- How to draw internal time-dependent features like the mass, area, entropy, trapped surfaces, etc?
- How is the Hawking radiation backreaction modeled in the spacetime?
- And so on...

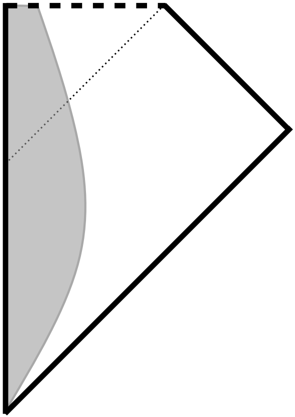
To remedy ambiguities we want a firmer mathematical basis.



Goal: Define a clear, mathematically rigorous spacetime, that adequately models the evaporation process, for which the diagram and physical features can be explicitly calculated.

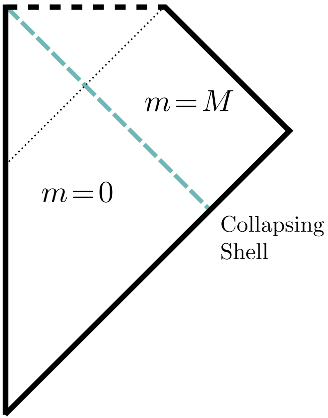
Plan:

- Introduce the class of models.
- Analyze the model and the diagrams.
- Extract the qualitative features of the spacetime.
- Consequences for unitarity? (Briefly!)

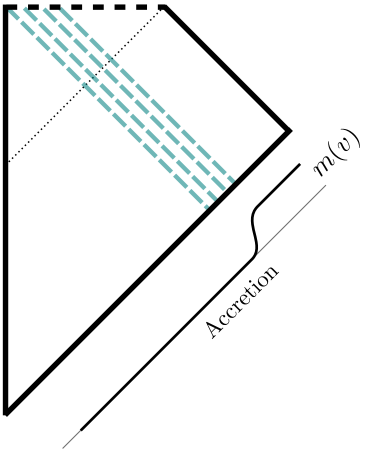


Let's build up a simple shell model of BH formation and evaporation.

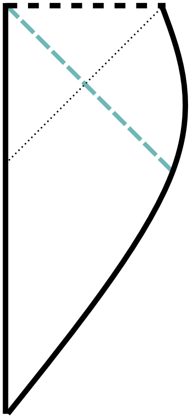
- Massless scalar field
- Slicing/gluing construction



BH formed by shell collapse.



Arbitrary accretion curve.
Discrete stepwise approximation.



Allowed metrics:

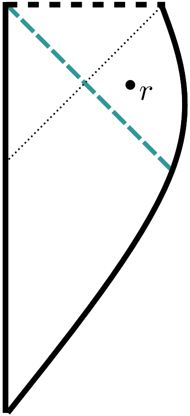
$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}dr^2 + r^2\,d\Omega^2.$$

Arbitrary *metric function* $f(r)$.

Examples:

Minkowski	$f(r) = 1,$
de Sitter	$f(r) = 1 - r^2/L^2,$
Anti de Sitter	$f(r) = 1 + r^2/L^2,$
Schwarzschild	$f(r) = 1 - 2m/r,$
Reissner-Nordstrom	$f(r) = 1 - 2m/r + Q^2/r^2,$
Hayward	$f(r) = 1 - 2mr^2/(r^3 + 2ml^2),$

... and combos like Hay-dS etc.



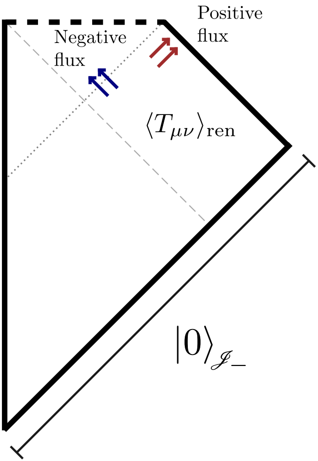
Metric function and mass function:

$$f(r) = 1 - \frac{2\,m(r)}{r}$$

Eigenvalues of $G^{\mu}{}_{\nu}$ in ON basis:

$$\rho = \frac{m'(r)}{4\pi r^2} \qquad p_{\Omega} = \frac{m''(r)}{8\pi r}$$

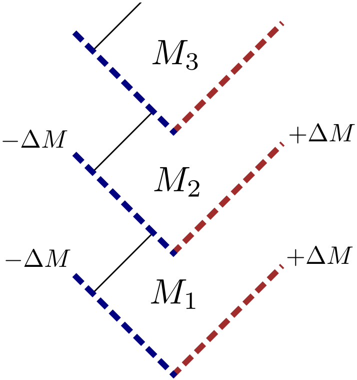
Radius r is physical, defined by area.



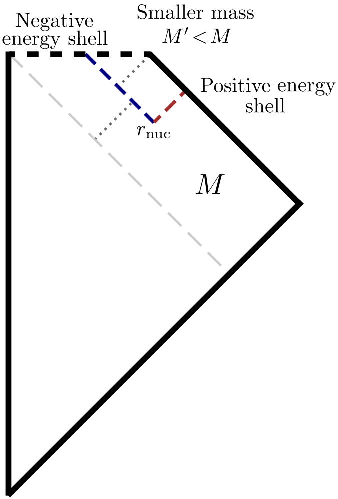
How to model evaporation?

Based on calculations of $\langle T_{\mu\nu} \rangle_{\mathrm{ren}}$ for massless scalar field.

- DFU 1976 *(Davies, Fulling, Unruh)*



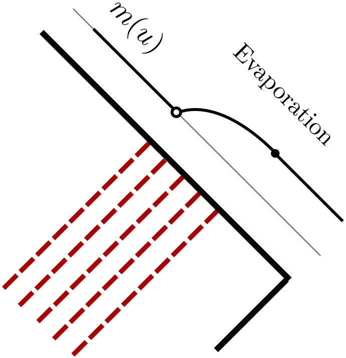
Gluing Schwarzschilds of decreasing mass naturally gives this $T_{\mu\nu}$ structure.



Model forces us into some constraints!

- Consistency $\implies$ $r_{\rm nuc} > 2m$.
- Energy conservation $\implies$ $r_{\rm nuc} \to 2m$.

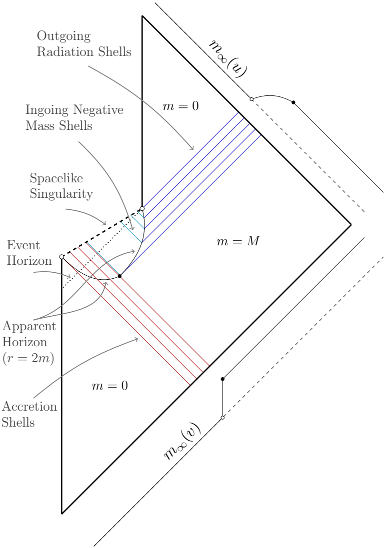
Radiation shells must be emitted from just outside AH at dynamical $r = 2M$.



Arbitrary evaporation curve.
Discrete stepwise approximation.

All together:

$$ds^2 = -\Big(1 - \tfrac{2m(u,v)}{r(u,v)}\Big)du\,dv + r(u,v)^2\,d\Omega^2$$



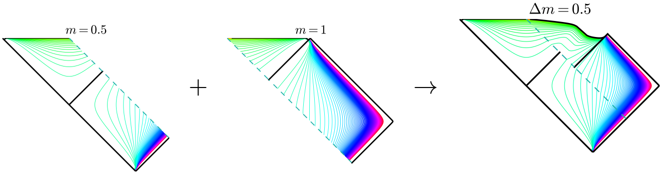
All together: $$ds^2 = -\left(1 - \frac{2m(u,v)}{r(u,v)}\right) du\, dv + r(u,v)^2\, d\Omega^2$$

So that's the schematic — now let's do it for real!

...and verify this was drawn correctly.

Gluing rules: (B-I formalism)

- Match intrinsic metric on the shell $\to$ radius matching.



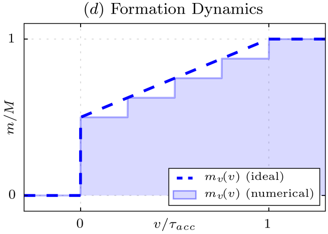
- Shell stress tensor in EF coords (ingoing/outgoing matters!)

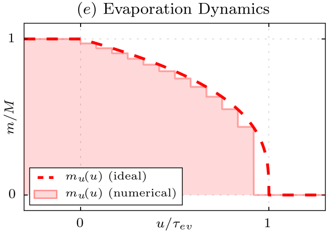
$$T_{\mu\nu} = \sigma\, n_\mu n_\nu\, \delta(w), \qquad \sigma = \pm \frac{\Delta m}{4\pi r^2}.$$

When shells cross: (evap nucleation points)

- Energy conservation $\implies$ DTR relation: $f_A(r_0) f_B(r_0) = f_C(r_0) f_D(r_0)$.

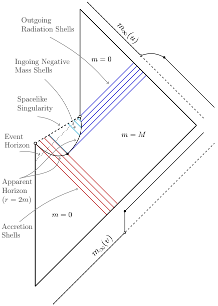
Accretion/Evaporation Dynamics:

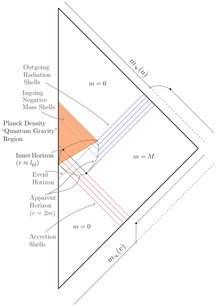




Schwarzschild and Hayward Interiors:

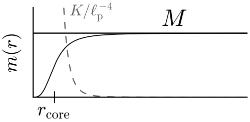
- Schwarzschild: Point mass. Infinite density singularity.
- Hayward: Not-quite-point mass. Tiny Planck density QG matter “core”.





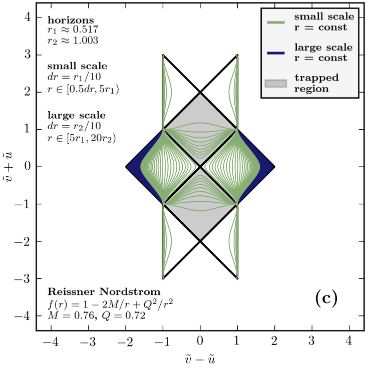
H/S metrics differ only in the region of super-Planckian Kretschmann curvature.

$$\ell_p \ll r_{\text{core}} \ll 2m, \quad r_{\text{core}} = (2m\ell_p^2)^{1/3}$$



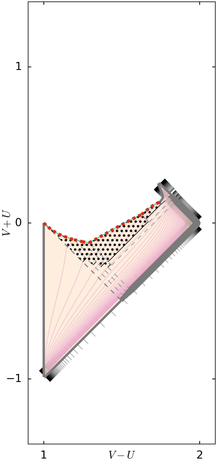
Generating diagrams explicitly:

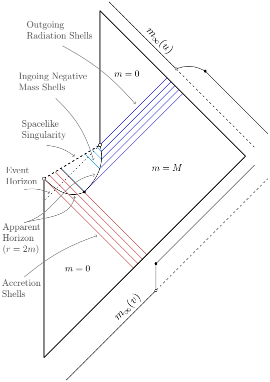
- Requires obtaining a global, compact, double-null coordinate system.
- Developed methods for arbitrary $f(r)$ with arbitrary number of horizons.
- See algorithms paper or xhorizon.



Numerical diagrams

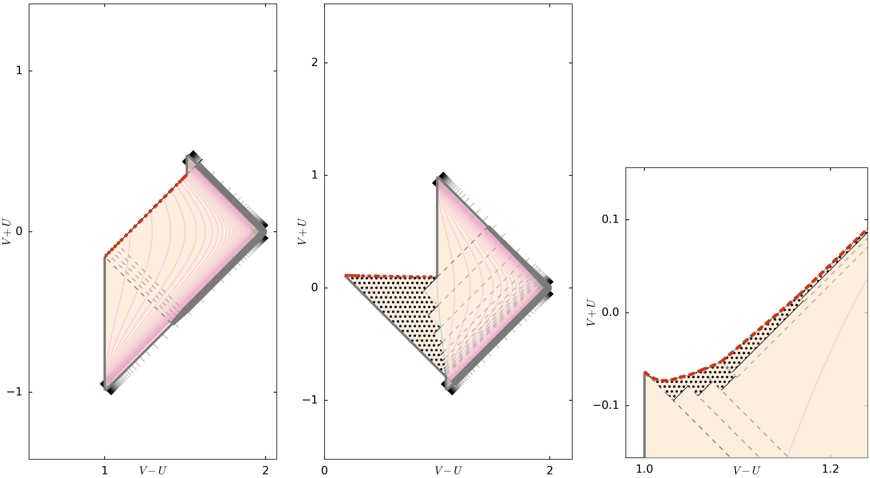
First: less realistic parameters for easier visibility.





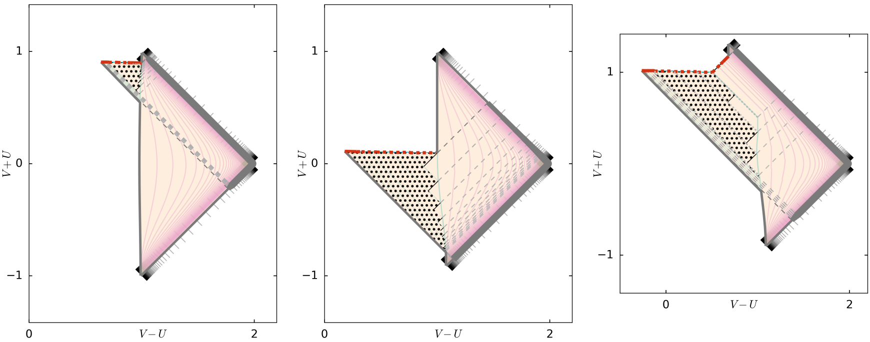
More realistic requires several views to piece together vastly different timescales.

Numerical diagrams: (Schwarzschild interior)



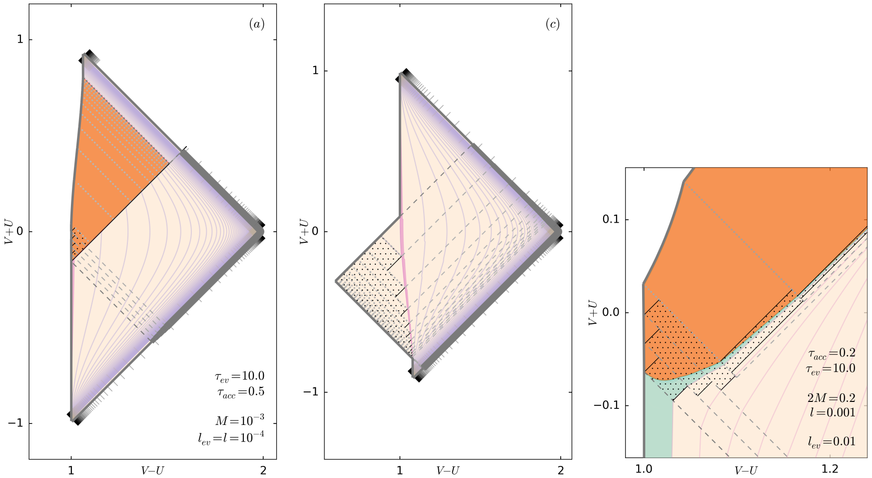
Perspectives: "Early", "Late", "Zoom".
Same diagrams (conformally related), squished in different ways.

Numerical diagrams: (Schwarzschild interior)



Perspectives: "Early", "Late", "During".
Same diagrams (conformally related), squished in different ways.

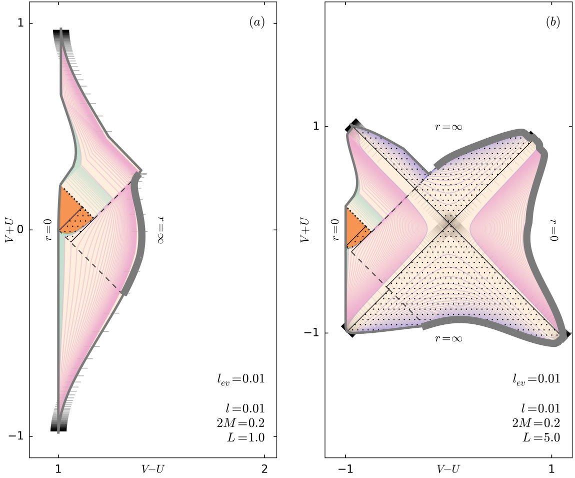
Numerical diagrams: (Hayward interior)



Perspectives: "Early", "Late", "Zoom".

Same diagrams (conformally related), squished in different ways.

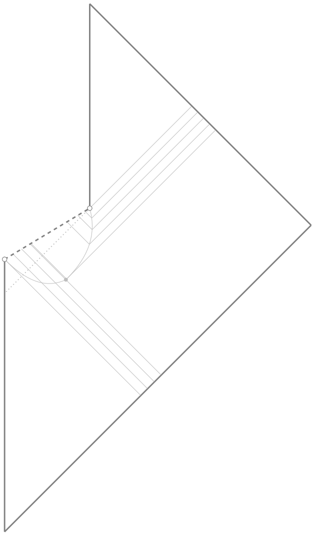
Numerical diagrams: (dS/AdS background)



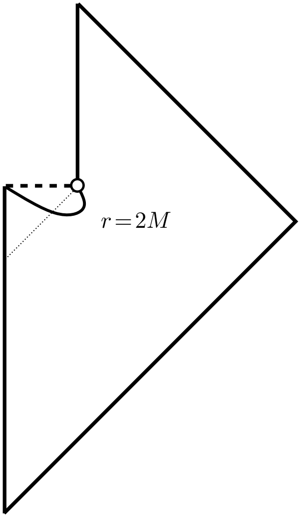
Takeaways?

Let's extract some qualitative features.

- $r = 2M$
- EH vs $2M$
- Hawking modes vs shell pairs
- Cauchy surfaces
- AH and trapping
- EC violations
- Mass inflation?
- "Real" BH spacetime?



Where is $r = 2M$?



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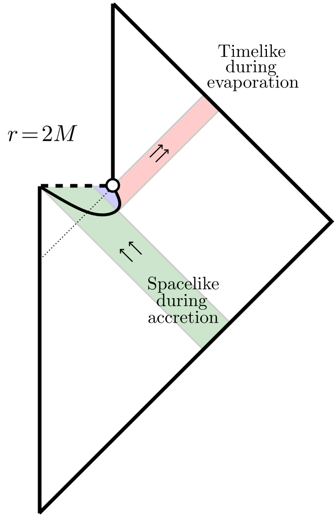
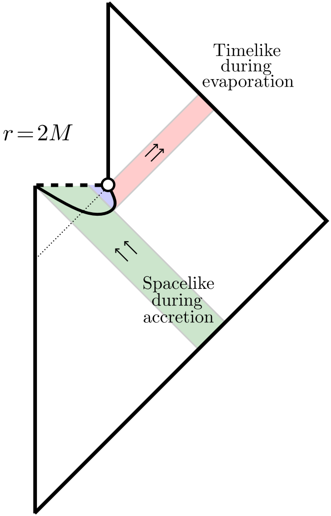


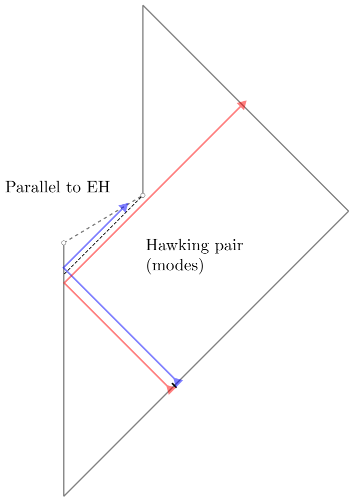
$r = 2M$
Timelike during evaporation
Spacelike during accretion

Event horizon vs Schwarzschild radius?

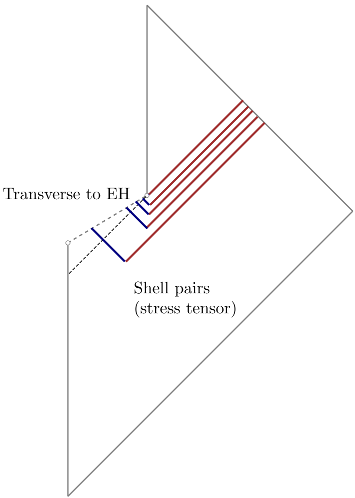
Not interchangeable terms.
Radiation emitted from $2m$ not EH.



Hawking modes

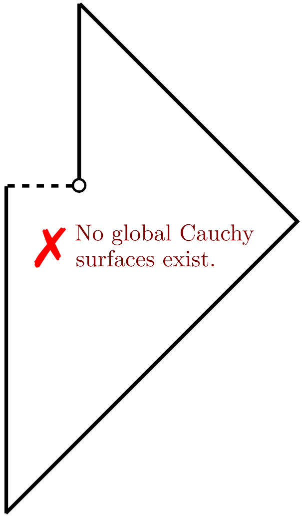


Stress tensor



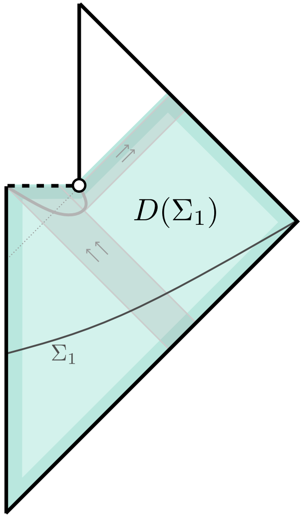
(Negative proper local energy vs negative Killing momentum.)

Where are Cauchy surfaces?



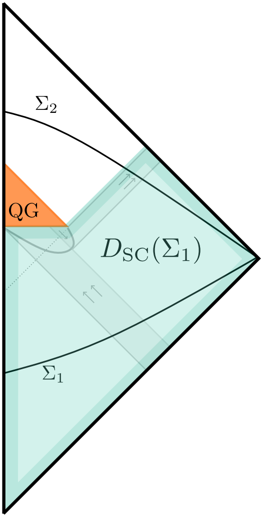
Globally hyperbolic subdomain $D(\Sigma_1)$ contains all evaporation physics.

Safer to work in this region for discussions of unitarity.



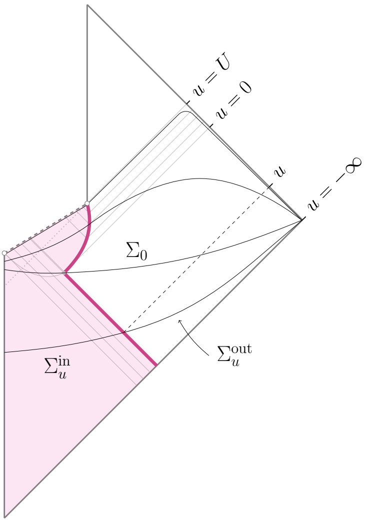
Regular restores GH classically.
But Σ_2 depends on QG physics.

Early region $D_{\mathrm{SC}}(\Sigma_1)$ same as singular case.

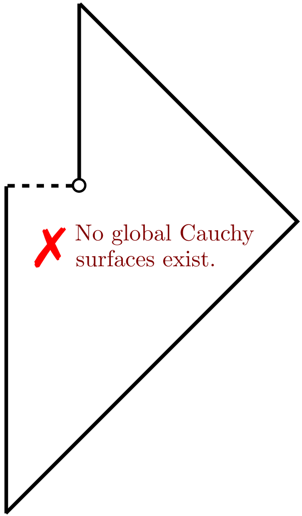


"Page Time" and "Evaporation Time" unitarity questions can be rigorously formulated completely on a foliation of this semiclassical region.

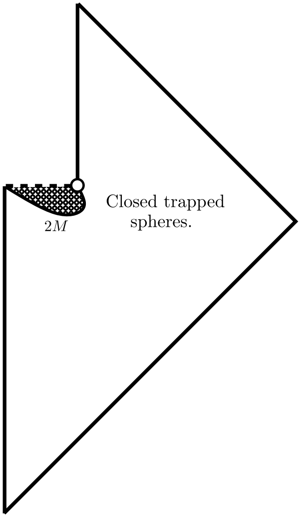
(arxiv:2012.07973)



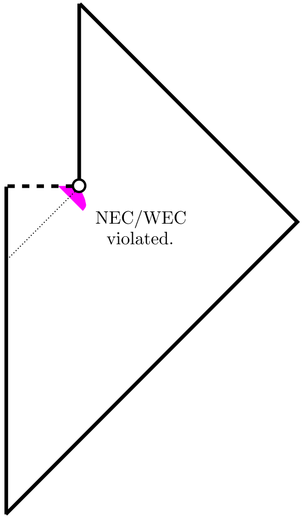
But: Relevant to “Long Term” unitarity question!
This diagram has $\mathscr{I}$ non-unitarity baked in.



Where are apparent horizons and trapped surfaces?



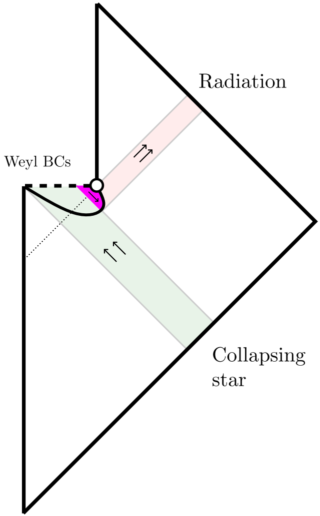
Where are energy conditions violated?



Follow the matter?

Hawking 1976:

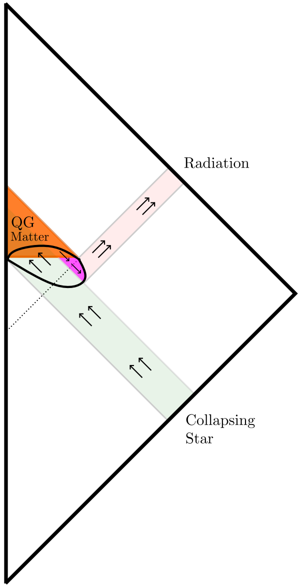
can regard the emitted particles as having come from the singularity inside the black hole and having tunnelled out through the event horizon on spacelike trajectories. Thus even an observer at



The stories we tell...

GR breaks down at Planck curvature?

• *Tiny QG dot grows and shrinks during accretion/evaporation.*



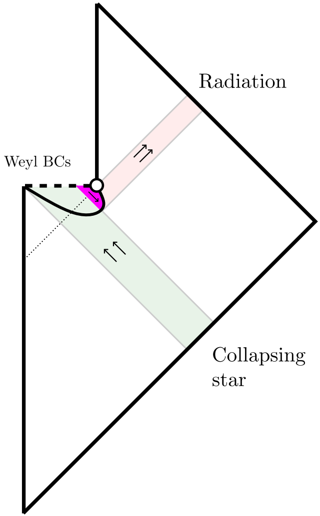
The stories we tell...

Classical GR holds at all scales?

• *Collapsing star departs universe at singularity, recorded in Weyl tensor. New matter emerges in radiation.*

Beyond vacuum addiction? Not just information...

BHs in space are made of "stuff"!



Finally, some concluding comments.

How generic is this model?

- Conclusions supported by most other similar studies like Hiscock, Hajicek, Vaidya-type models, Ashtekar review, and others.
- So seems quite generic.
  That said, I'm not sure. Further study?

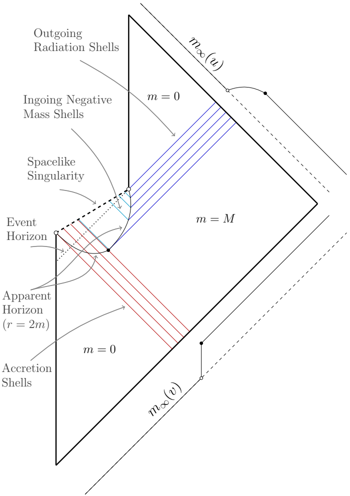
Warning! 1+1d shell models very different.

- Intrinsic metric on shell is trivial in 1+1d.
- 3+1d radial shells have an inside/outside and are collapsing/expanding, physically.

Spin and charge?

- Drastically alters Schwarzschild, but not Hayward, picture.

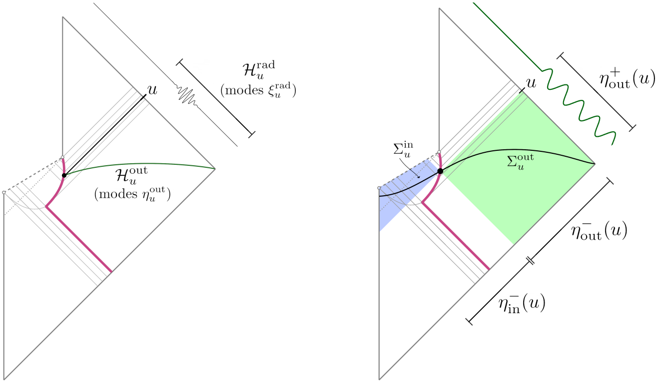
Bottom line: *To draw something qualitatively different than this, one should probably specify their model or show such a 3+1d model exists.*



Unitarity?

Aim was to continue with the "explicitly constructive" approach, to be unambiguous about Hilbert spaces defining the information problem.

arxiv:2012.07973



Physical principles of unitarity.

Quantum gravity unitarity

- *Unitary QG Hilbert space*

Semiclassical unitarity

- *If Σ_1 and Σ_2 have the same domain of dependence, unitary evolution of the SC quantum fields.*

Common unitarity statements.

Long term unitarity

- *Unitary scattering matrix of semiclassical modes from past to future null infinity.*

Evaporation time unitarity

- *Is the Hawking radiation in a pure state when evaporation ends?*

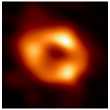
Page time unitarity

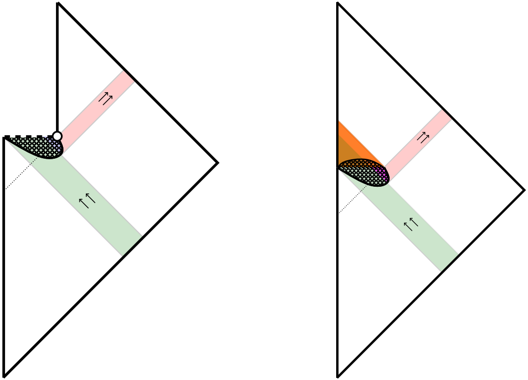
- *Does the entropy of Hawking radiation decrease at late times during evaporation?*

Conclusions: The common statements, if they refer to $\mathcal{H}^{\rm SC}_{\rm rad}$, can all be violated *without* violating either physical principle. Expect Page curve for $\mathcal{H}^{\rm QM}_{\rm rad}$ but not $\mathcal{H}^{\rm SC}(\Sigma_{\rm out})$. (Agrees with holography if in slightly different language.)

See the unitarity paper arxiv:2012.07973 for more.

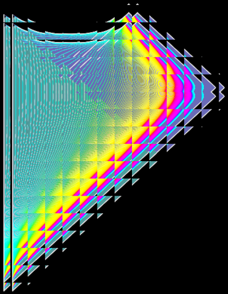
How to define BHs if we don't know quantum gravity?





What do these have in common?

- Trapped region feeding into super-Planckian curvature region.
- EH $\rightarrow$ Doom Horizon: Doomed to super-Planckian quantum soup.



xhorizon.readthedocs.io

Thank you!!

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Amita Kuttner

Anthony Aguirre

(Algorithms, 2018) arxiv:1802.02263
(Evaporation, 2019) arxiv:1907.04879
(Unitarity, 2021) arxiv:2012.07973

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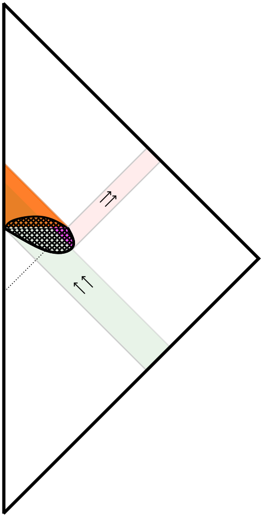
$S \leftarrow A/4$ since 2022.

Mass inflation?

Inner horizon inside core.



The "real" spacetime of an evaporating BH?