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Response of an accelerated detector coupled to the stress–energy tensor

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Abstract. We consider the response of accelerated detectors which couple to the components of the stress-energy tensor of the field linearly and locally. We show that these detectors become excited if put on an accelerated trajectory and evaluate the rate of excitation for the simple case when the coupling is to the trace of $T^i{}_k$. These results arise from the fact that detectors respond to the power spectrum of the two-point function $\langle 0|T_{ik}(x)T_{lm}(y)|0\rangle$ rather than to $\langle 0|T_{ik}|0\rangle_{\text{reg}}$. The latter quantity vanishes in the accelerated frame but the former does not. The consequences of the result are discussed.

1. Introduction and motivation

In order to measure physical variables related to a field, we need a system ('detector') which couples to the field in some form. The response of such a detector will depend on: (i) the state of the field, (ii) the nature of the coupling and (iii) the state of motion of the detector.

One of the simplest such models involves a massless scalar field $\phi(x)$ and a detector coupled locally and linearly to the field variable $\phi(x)$. The interaction Hamiltonian for such a system will be

$$H_I = \int d\tau\, \mu(\tau)\phi(\tau, x^i(\tau)) \tag{1}$$

where μ is a variable characterising the detector and $x^i(\tau)$ is the trajectory of the detector with τ being the proper time. We shall call this system a 'field-coupled detector' (FCD).

Let us assume that the scalar field is in the vacuum state $|0\rangle_{\text{I}}$, defined in the usual way in an inertial Minkowski coordinate system. Since we have now specified both the state of the quantum field and the coupling of the detector, the response of the detector depends only on the trajectory of the detector: $x^i(\tau)$. It is well known that the detector will not 'click' (by which we mean, it will not make a transition from an internal energy state $|E_0\rangle$ to a state $|E_1\rangle$ with $E_1 > E_0$; the term 'click' is used in this sense throughout this paper) while in an inertial motion but will click while in a

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uniformly accelerated motion (Unruh 1976, DeWitt 1979). In fact, the detector will click in a wide variety of non-inertial motions of which the uniformly accelerated trajectory (UAT) is just a special case (for an explicit construction of such trajectories see Letaw (1981) and Padmanabhan (1982)). The rate of excitation of the detector is determined by the power spectrum of the Wightman function

$$R(\omega; x^i(\tau)) = \int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau} \langle 0|\varphi(x(s+\tau))\phi(x(s))|0\rangle \tag{2}$$

which can be independent of s for a large class of motions (for details, see the references cited above).

Given the trajectory of the detector $x^i(\tau)$, one can set up a proper non-inertial coordinate system S for the detector. For a large class of $x^i(\tau)$—which includes UAT—this non-inertial frame will be stationary, i.e. the components of the metric tensor in S will be independent of the proper time τ. One can apply quantum field theory in S using mode functions which are positive- (or negative-) frequency components with respect to τ. This allows one to define creation and annihilation operators (a_ω^+, a_ω for each mode) and a vacuum state $|0\rangle_s$ in the non-inertial frame. The expectation value of the operator $(a_\omega^+ a_\omega)$ in the Minkowski vacuum

$$N(\omega) \equiv {}_{\rm I}\langle 0|a_\omega^+ a_\omega|0\rangle_{\rm I} \tag{3}$$

may not (in general) vanish. In particular, for UAT the quantities $R(\omega)$ and $N(\omega)$ happen to be proportional to each other

$$R(\omega) \propto N(\omega) \qquad \text{for UAT.} \tag{4}$$

This proportionality has led to somewhat incorrect and confusing ideas in the literature. Because of (4) one is tempted to conclude that: (i) the inertial vacuum is 'populated by Rindler particles' with a spectrum $N(\omega)$, (ii) the FCD of (1) is a 'particle' detector and (iii) that the FCD clicks in UAT *because* it detects the Rindler particles in $|0\rangle_{\rm I}$. Each of the above conclusions contains hidden subtleties and demands closer scrutiny. (Part of the problem, of course, is just semantics and can be eliminated by clear mathematical definitions; we are only concerned with subtleties relevant to physics.)

To begin with, the extremely suggestive proportionality (4) does not hold true in general. There exists a large class of trajectories in which $R(\omega)$ and $N(\omega)$ have no interrelationship (see Letaw 1981, Padmanabhan 1982). In particular there are trajectories with

$$R(\omega) \neq 0 \qquad N(\omega) = 0 \tag{5}$$

(the uniformly rotating frame being one such example). In other words, the detector will click even though the $|0\rangle_{\rm I}$ does not contain any of the 'non-inertial particles' represented by the operator $(a^+ a)$. This fact shows that the conclusion (iii) above is incorrect. It is not the 'presence of particles' (i.e. existence of non-zero $N(\omega)$) which is the *reason behind* the clicking of the detector.

The conclusions (i) and (ii) are trickier. To settle (i), we have to define precisely the phrase 'populated by Rindler particles'. This is a question of definition, and one possible reasonable definition is that a state $|\psi\rangle$ contains particles of a particular nature if $\langle\psi|a^+ a|\psi\rangle$ is non-zero, where a^+ and a are suitably defined creation and annihilation operators. This is, for example, the definition that a condensed matter physicist would use to settle the existence (or otherwise) of quasiparticles in a state. If we accept this definition, then the inertial vacuum *is* populated by Rindler 'particles' but *not* by 'particles' based on a rotating frame quantisation. (Remember, however, that both rotating *and* uniformly accelerating detectors will click in $|0\rangle_I$; as mentioned before there is no one-to-one correspondence between detector response ($R(\omega)\neq 0$) and the presence of particles ($N(\omega)\neq 0$).)

Once we have settled on $N(\omega)\neq 0$ as our definition for 'existence of particles', it follows that statement (ii) is somewhat misleading. The FCD is not a 'particle' detector in the sense that it does not *in general* measure $N(\omega)$, although it does so while in the inertial trajectory and in UAT.

We may, then, ask: why does this 'detector' respond, and what exactly does it detect? The answer to the first question was always intuitively clear (see, e.g., DeWitt (1979, p 694 third paragraph) and Birrell and Davies (1982, p 55, second paragraph)) and was explicitly demonstrated by one of the present authors (Padmanabhan 1985). It was shown using an explicit model for the accelerating mechanism that: (i) the accelerating source supplies energy to the detector and (ii) part of this energy is utilised for the detector to make an internal transition from $|E_0\rangle$ to $|E_1\rangle$ ($E_1 > E_0$) and the rest of the energy is radiated as field quanta in some mode $\boldsymbol{k}$, changing the state of the field from $|0\rangle_I$ to $|1_k\rangle$. This is how a Minkowski observer will interpret the process (see Padmanabhan 1985). From the point of view of a Minkowski observer, the 'detector' is not detecting anything but is radiating; hence the question 'what is it detecting?' has no meaning in the Minkowski frame (compare this, for example, with the discussion in Unruh and Wald (1984) in which a somewhat different interpretation is presented).

The story is different in the accelerated frame. From (2) it is clear that the detector is responding to the power spectrum of the vacuum fluctuations of the field $\phi(x)$. In other words FCD acts as a 'fluctometer' (Candelas 1980, Candelas and Sciama 1983) and responds to the fluctuations in $\varphi(x)$. We know that, even though the expectation value ${}_I\langle 0|\phi(x)|0\rangle_I$ vanishes, the fluctuations in ϕ, characterised by

$$ {}_I\langle 0|\phi(t_2,\boldsymbol{x})\phi(t_1,\boldsymbol{x})|0\rangle_I = -(1/4\pi^2)(t_2-t_1-\mathrm{i}\varepsilon)^{-2} \tag{6}$$

do not. The Fourier transform of (6)

$$ R(\omega) = -\int_{-\infty}^{+\infty} \mathrm{d}t \frac{\mathrm{e}^{-\mathrm{i}\omega t}}{4\pi^2(t-\mathrm{i}\varepsilon)^2} = -\frac{\omega}{2\pi}\,\theta(-\omega) = 0 \tag{7}$$

vanishes for $\omega > 0$ when evaluated along an inertial trajectory. However $R(\omega)$ is not a covariant object and can be non-zero when evaluated along a different trajectory. It is this vacuum noise of $\phi(x)$ which is seen by our 'fluctometer'.

Considering the importance of the issue, it is worthwhile to reinforce the above conclusions by different lines of argument. We shall do that in this paper by considering the response of detectors which couple linearly to the components of the stress-energy tensor $T^i{}_k$ of the scalar field via the interaction Hamiltonian of the type

$$ H_I = \int \mathrm{d}\tau\, \mu^i{}_k(\tau) T^k{}_i[x(\tau)] \tag{8}$$

where $\mu^i{}_k$ is a suitable detector variable. We ask the question: will this 'energy-coupled detector' (ECD) click if put on an accelerated trajectory?

One may be tempted to answer that it will not, based on the following consideration. It has been repeatedly stressed in the literature that the regularised vacuum expectation value (VEV) of $T^i{}_k$ is a covariant object (assuming, of course, that a suitable covariant regularisation scheme is resorted to). Therefore, if ${}_I\langle 0|T^i{}_k|0\rangle_{I,\mathrm{reg}}$ vanishes in one frame (in the Minkowski frame, say) then it must vanish in all frames and especially in the Rindler frame. Therefore, the detector will not click.

This argument, however, is fallacious. It is perfectly true that ${}_{\rm I}\langle 0|T^i{}_k|0\rangle_{\rm I,reg}$ is a covariant object and that it vanishes both in the inertial frame and in the accelerated frame. But the response of the ECD has nothing to do with the VEV ${}_{\rm I}\langle 0|T^i{}_k|0\rangle_{\rm I,reg}$! What the ECD will respond to is the power spectrum of the vacuum *fluctuations* in $T^i{}_k$, i.e.

$$\mathscr{G}^{ij}{}_{kl}(\omega)=\int_{-\infty}^{+\infty}\mathrm{d}\tau\,\mathrm{e}^{-\mathrm{i}\omega\tau}{}_{\rm I}\langle 0|T^i{}_k(x(s+\tau))\,T^j{}_l(x(s))|0\rangle_{\rm I}. \tag{9}$$

There is no reason for ${}_{\rm I}\langle 0|T^i{}_kT^j{}_l|0\rangle_{\rm I}$ to vanish even though ${}_{\rm I}\langle 0|T^i{}_k|0\rangle_{\rm I}$ may vanish. This can be, in fact, seen very clearly by going back to our FCD. The VEV of $\phi(x)$, ${}_{\rm I}\langle 0|\phi(x)|0\rangle_{\rm I}$ is certainly a covariant object and it certainly vanishes in both inertial and accelerated frames, but this does not prevent a detector coupled linearly to $\phi(x)$ from clicking as long as ${}_{\rm I}\langle 0|\phi(x)\phi(y)|0\rangle_{\rm I}$ has a non-zero power spectrum! The situation is identical in the case of the ECD where ϕ is replaced by $T^i{}_k$.

Thus there is no *a priori* reason for the detector coupled to $T^i{}_k$ not to click when accelerated. We shall work out the details in §§ 2 and 3 and will show that the ECD *does* click when accelerated. This result demonstrates that the 'detectors' like FCD or ECD are not actually responding to simple physical observables but are measuring the fluctuations in these observables. This conclusion has some interesting implications which we will discuss in the last section.

2. The response of ECD

Consider an ECD described by the coupling in (8). We are interested in the transition amplitude between an initial state

$$|I\rangle=|E_0\rangle\otimes|0\rangle \tag{10}$$

and a final state

$$|F\rangle=|E_1\rangle\otimes|\psi\rangle \tag{11}$$

where $|E_0\rangle$ and $|E_1\rangle$ characterise the internal energy states of the ECD (with $E_1>E_0$), $|0\rangle$ is the Minkowski vacuum state for the field, and $|\psi\rangle$ is the final state of the field. To the lowest order in the coupling the transition amplitude is given by

$$\begin{aligned}A(E_1,E_0;\psi,0)&=\int_{-\infty}^{+\infty}\mathrm{d}\tau\,\langle F|\mu^i{}_k(\tau)T^k{}_i(x(\tau))|I\rangle\\&=\int_{-\infty}^{+\infty}\mathrm{d}\tau\,\langle E_1|\mu^i{}_k(\tau)|E_0\rangle\langle\psi|T^k{}_i(x(\tau))|0\rangle\end{aligned} \tag{12}$$

where $x^i(\tau)$ is the trajectory of the detector and τ is the proper time. Making the usual assumption that (see, e.g., DeWitt 1979, equation (14-9)),

$$\mu^i{}_k(\tau)=\mathrm{e}^{\mathrm{i}H_0\tau}\,\mu^i{}_k(0)\,\mathrm{e}^{-\mathrm{i}H_0\tau} \tag{13}$$

where H_0 is the Hamiltonian operator governing the internal dynamics of the detector, we can write ($\omega\equiv E_1-E_0>0$)

$$A(E_1,E_0;\psi,0)=\langle E_1|\mu^i{}_k(0)|E_0\rangle\int_{-\infty}^{+\infty}\mathrm{d}\tau\,\mathrm{e}^{\mathrm{i}\omega\tau}\langle\psi|T^k{}_i[x(\tau)]|0\rangle. \tag{14}$$

Let us now consider the quantity $\langle\psi|T^k{}_i(x)|0\rangle$. Since

$$T^i{}_k = \partial^i\phi\partial_k\phi - \tfrac{1}{2}\delta^i{}_k(\partial^a\phi)(\partial_a\phi) \tag{15}$$

is quadratic in ϕ (and hence in creation and annihilation operators), $\langle\psi|T^k{}_i(x)|0\rangle$ can be non-zero only when $|\psi\rangle$ is the vacuum state $|0\rangle$ or when $|\psi\rangle$ is a two-particle state $|1_p, 1_q\rangle$ labelled by the momenta $(\boldsymbol{q}, \boldsymbol{p})$ (of course, $\boldsymbol{p}=\boldsymbol{q}$ is included as a special case). However, the VEV $\langle 0|T^k{}_i(x)|0\rangle$ need not be considered. Formally, this is a divergent expression but is a constant independent of x:

$$\langle 0|T^i{}_k(x)|0\rangle = \langle 0|\,\mathrm{e}^{\mathrm{i}\hat{P}x}T^i{}_k(0)\,\mathrm{e}^{-\mathrm{i}\hat{P}x}|0\rangle = \langle 0|T^i{}_k(0)|0\rangle. \tag{16}$$

(We have merely used the translation invariance of the vacuum.) Such a term will contribute to the integral in (14) only a $\delta(\omega)$ term which vanishes because we are assuming $\omega>0$. Therefore the contribution to (14) arises from the $\langle 1_p, 1_q|T^i{}_k(x)|0\rangle$ term. Using the mode expansion

$$\phi(x) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{(2\omega_k)^{1/2}}(a_k\,\mathrm{e}^{-\mathrm{i}kx} + a_k^+\,\mathrm{e}^{\mathrm{i}kx}) \tag{17}$$

in (15) and evaluating $\langle 1_p, 1_q|T^i{}_k(x)|0\rangle$ one obtains

$$\langle 1_p, 1_q|T^i{}_k(x)|0\rangle = -\frac{1}{(2\omega_p)^{1/2}}\frac{1}{(2\omega_q)^{1/2}}(p^iq_k + p_kq^i - \delta^i{}_k(p\cdot q))\,\mathrm{e}^{\mathrm{i}(p+q)x}. \tag{18}$$

(The state $|1_p, 1_q\rangle$ is *defined* to be $a_p^+a_q^+|0\rangle$ in all expressions. It differs from the conventional two-particle state by a normalisation factor $1/\sqrt{2}$ when $\boldsymbol{p}=\boldsymbol{q}$; it is easier to work with $|1_p, 1_q\rangle$.) Instead of performing the calculation using (17) and (15) one can also guess most of the contents of (18) by the following argument. Note that

$$\langle 1_p, 1_q|T^i{}_k(x)|0\rangle = \langle 1_p, 1_q|\,\mathrm{e}^{\mathrm{i}\hat{P}x}T^i{}_k(0)\,\mathrm{e}^{-\mathrm{i}\hat{P}x}|0\rangle = \mathrm{e}^{\mathrm{i}(p+q)x}\langle 1_p, 1_q|T^i{}_k(0)|0\rangle. \tag{19}$$

The tensor $\langle 1_p, 1_q|T^i{}_k(0)|0\rangle$ must be constructed from a quadratic form made of p^i and q^k such that: (i) it is symmetric under the (p, q) interchange, (ii) it changes sign when the sign of p^i or q^k is reversed and (iii) it has zero dot product with $(p+q)^i$—so that the conservation of $T^i{}_k(x)$ is assured. These conditions reduce the expression to a form proportional to $(p^iq_k + p_kq^i - \delta^i{}_k(p\cdot q))$. (The proportionality constant, however, has to be fixed by explicit computation giving the form in (18)!)

Using (18) and (14) we can write the transition amplitude to be

$$A(\omega; \boldsymbol{p}, \boldsymbol{q}) = -\alpha^k{}_i\int_{-\infty}^{+\infty}\mathrm{d}\tau\,\mathrm{e}^{\mathrm{i}\omega\tau}\frac{1}{(2\omega_p)^{1/2}}\frac{1}{(2\omega_q)^{1/2}}(p^iq_k + p_kq^i - \delta^i{}_k(p\cdot q))\,\mathrm{e}^{\mathrm{i}(p+q)x} \tag{20}$$

where we have used the notation $\alpha^k{}_i = \langle E_1|\alpha^k{}_i(0)|E_0\rangle$. We are, however, interested in the *probability* P that the ECD makes the transition from E_0 to E_1 *irrespective* of the final state of the field. Therefore we have to sum $|A|^2$ over the final field states to obtain P:

$$P(\omega) = \int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\int\frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}|A(\omega; \boldsymbol{p}, \boldsymbol{q}, 0)|^2 \tag{21}$$

$$= \int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2\omega_p}\int\frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}\frac{1}{2\omega_q}$$

$$\times\int\mathrm{d}\tau_1\,\mathrm{d}\tau_2\,\exp[\mathrm{i}\omega(\tau_1-\tau_2)]\exp[\mathrm{i}(p+q)(x_1-x_2)]\alpha^k{}_i\alpha^m{}_nh^i{}_kh^n{}_m \tag{22}$$

where

$$h^a{}_b \equiv (p^a q_b + p_b q^a - \delta^a{}_b (p\cdot q)) \tag{23}$$

and

$$x_1^i = x^i(\tau_1) \qquad x_2^i = x^i(\tau_2). \tag{24}$$

Interchanging the orders of integration we can write $P(\omega)$ as

$$P(\omega) = \int \mathrm{d}\tau_1\,\mathrm{d}\tau_2 \exp[\mathrm{i}\omega(\tau_1-\tau_2)]\alpha^k{}_i \alpha^m{}_n \mathscr{G}^{in}{}_{km}(x_2-x_1) \tag{25}$$

where

$$\mathscr{G}^{in}{}_{km}(x_2-x_1) = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\int\frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}\frac{1}{2\omega_p}\frac{1}{2\omega_q} h^i{}_k h^n{}_m \exp[\mathrm{i}(p+q)(x_1-x_2)]$$
$$\equiv \langle 0|T^n{}_m(x_2)T^i{}_k(x_1)|0\rangle_{\text{reg}}. \tag{26}$$

This integral can be evaluated by a series of steps. First, put $x_2 = x_1 + x$ and consider the quantity

$$F^{inkm}(x) \equiv \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\int\frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}\frac{1}{2\omega_p}\frac{1}{2\omega_q} p^i p^n q^k q^m \exp[-\mathrm{i}(p+q)x] \tag{27}$$

$$= \left(\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2\omega_p} p^i p^n\, \mathrm{e}^{-\mathrm{i}px}\right)\left(\int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}\frac{1}{2\omega_q} q^k q^m\, \mathrm{e}^{-\mathrm{i}qx}\right). \tag{28}$$

Now

$$\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2|\boldsymbol{p}|} p^i p^n\, \mathrm{e}^{-\mathrm{i}px} = -\frac{\partial^2}{\partial x_i\,\partial x_n}\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2|\boldsymbol{p}|}\mathrm{e}^{-\mathrm{i}px}$$
$$= \frac{\partial^2}{\partial x_i\,\partial x_n}\left(\frac{1}{4\pi^2}\frac{1}{(x^0-\mathrm{i}\varepsilon)^2-|\boldsymbol{x}|^2}\right) \equiv \frac{\partial^2}{\partial x_i\,\partial x_n} W(s^2) \tag{29}$$

where $s^2 = x^i x_i$ and $W = (4\pi^2 s^2)^{-1}$ with the understanding that there is an $(\mathrm{i}\varepsilon)$ in x^0. Since (with $W' = \mathrm{d}W/\mathrm{d}s^2$, etc)

$$\frac{\partial^2}{\partial x_i\,\partial x_n} W(s^2) = \frac{\partial}{\partial x_i}\left[\left(\frac{\mathrm{d}W}{\mathrm{d}s^2}\right)2x^n\right]$$
$$= 2W'(s^2)g^{ni} + 4W'' x^i x^n = (2W/s^4)(4x^i x^n - g^{ni}s^2) \tag{30}$$

we can write (28) as

$$F^{inkm}(x) = (4W^2/s^8)(4x^i x^n - g^{ni}s^2)(4x^k x^m - g^{km}s^2). \tag{31}$$

The $h^{ik}h^{nm}$ of (26) is just

$$h^{ik}h^{nm} = p^i p^n q^k q^m + p^k p^n q^i q^m - g^{nm}(p\cdot q)p^i q^k$$
$$- g^{ik}(p\cdot q)p^n q^m + \tfrac{1}{2}g^{ik}g^{nm}(p\cdot q)^2 + (\text{terms } p \leftrightarrow q) \tag{32}$$

so that the integral in (26) can be expressed in terms of the combinations of $F^{in\,km}(x)$. Performing this algebra we obtain ($F \equiv g_{ab}g_{cd}F^{acbd}$):

$$\mathscr{G}^{inkm}(x) = 2(F^{inkm} + F^{knim} - g^{nm}g_{ab}F^{iakb} - g^{ik}g_{ab}F^{anbm} + \tfrac{1}{2}g^{ik}g^{nm}F)$$
$$= (8W^2/s^8)[32x^i x^n x^k x^m + s^4(g^{in}g^{km} + g^{kn}g^{im} + 4g^{ik}g^{nm})$$
$$- 4s^2(g^{in}x^k x^m + g^{km}x^i x^n + g^{kn}x^i x^m$$
$$+ g^{im}x^k x^n + 2g^{nm}x^i x^k + 2g^{ik}x^n x^m)]$$
$$\equiv (8W^2/s^8)[A^{inkm}(x) + s^4 C^{inkm}(x) - s^2 B^{inkm}(x)]. \tag{33}$$

To obtain the response of the detector, one must specify the form of the detector variable $\alpha_{ik} = \langle E_1|\alpha_{ik}(0)|E_0\rangle$. Using the notation $A(x) = \alpha_{ik}\alpha_{nm}A^{inkm}$, etc, equation (25) becomes (with $\tau_2 = \tau_1 + \tau$)

$$P(\omega) = \int d\tau_1 \int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau}\left(\frac{8W^2}{s^8}(A - s^2B + s^4C)\right)$$

$$= \frac{1}{2\pi^4}\int d\tau_1 \int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau}\left(\frac{A - Bs^2 + Cs^4}{s^{12}}\right). \tag{34}$$

The quantity $(A - Bs^2 + Cs^4)/s^{12}$ depends only on the vector $x^i(\tau_2) - x^i(\tau_1)$, but in general this expression will depend on both τ_1 and τ_2 rather than just on $\tau_2 - \tau_1 = \tau$. (s^2 in the accelerated frame depends only on $\tau_2 - \tau_1$, but A, B, C can depend in general on both τ_1 and τ_2.) Therefore the quantity

$$R(\omega, \tau_1) \equiv \frac{1}{2\pi^4}\int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau}\left(\frac{A - Bs^2 + Cs^4}{s^{12}}\right) \tag{35}$$

should be interpreted as the (time-dependent) rate of transition from $|E_0\rangle$ to $|E_1\rangle$. In the special case where R is independent of τ_1 this expression will give a steady rate of transition. (The interpretation of R as a rate of transition is somewhat tricky when it depends on τ_1; for a careful discussion of this point see, e.g., Letaw (1981) and Birrell and Davies (1982). Our main conclusions, however, are independent of this interpretational issue.) We shall now see what can be said about $R(\omega, \tau_1)$ under various circumstances.

3. Specific examples of ECD

In order to demonstrate explicitly the behaviour of accelerated ECD we shall consider a simple specific example. Later we shall consider the more general situation.

The simple example we will take is the one in which $\mu^i{}_j$ has the form

$$\mu^i{}_j(\tau) = \mu(\tau)\delta^i{}_j \tag{36}$$

so that the detector actually couples to the trace of the stress tensor $T^a{}_a(x)$:

$$H_I = \int \mu(\tau)T(x(\tau))\, d\tau. \tag{37}$$

The form in (36) has the important advantage of being invariant—having the same form in both inertial and accelerated frames. The response of this detector is decided by the function $\langle 0|T(x_2)T(x_1)|0\rangle$ which can be obtained from our general expression for $\mathscr{G}^{inkm}$ by suitable contractions:

$$\langle 0|T(x_2)T(x_1)|0\rangle = \mathscr{G}^{ab}{}_{ab} = \frac{8W^2}{s^8}(24s^4) = \frac{12}{\pi^4}\frac{1}{s^8}. \tag{38}$$

The rate of transition of the detector, given by (35), becomes

$$R(\omega, \tau_1) = \frac{12\alpha^2}{\pi^4}\int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau}\left(\frac{1}{s^8}\right) = R(\omega) \tag{39}$$

(where $\alpha = \langle E_1|\mu(0)|E_0\rangle$).

We notice that the rate R is independent of τ_1, reminiscent of the situation with the FCD.

Consider first an ECD in an inertial trajectory $x=$ constant. We do not expect it to click and, in fact, it does not:

$$R_{\text{inertial}}(\omega)=\frac{12\alpha^2}{\pi^4}\int_{-\infty}^{+\infty}\mathrm{d}\tau\frac{\mathrm{e}^{-\mathrm{i}\omega\tau}}{(\tau-\mathrm{i}\varepsilon)^8}=0 \qquad \text{for } \omega>0. \tag{40}$$

(For $\omega>0$ the integral can be evaluated by closing the contour on the lower complex plane; the pole is at $\tau=\mathrm{i}\varepsilon$ on the upper plane.)

Next consider the uniformly accelerated trajectory: $x=g^{-1}\cosh g\tau$, $t=g^{-1}\sinh g\tau$, $y=z=0$. It is easy to show that

$$s^2(x(\tau_2),x(\tau_1))=(2g^{-1})^2\sinh^2\tfrac{1}{2}g(\tau_2-\tau_1-\mathrm{i}\varepsilon)=\frac{4}{g^2}\sinh^2\tfrac{1}{2}g(\tau-\mathrm{i}\varepsilon) \tag{41}$$

so that $R(\omega)$ becomes

$$R_{\text{UAT}}(\omega)=\frac{12\alpha^2}{\pi^4}\int_{-\infty}^{+\infty}\mathrm{d}\tau\frac{\mathrm{e}^{-\mathrm{i}\omega\tau}}{(2/g)^8\sinh^8[g(\tau-\mathrm{i}\varepsilon)/2]}$$
$$=\frac{3\alpha^2g^7}{32\pi^4}\int_{-\infty}^{+\infty}\mathrm{d}l\frac{\mathrm{e}^{-\mathrm{i}\nu l}}{\sinh^8(l-\mathrm{i}\varepsilon)} \tag{42}$$

where we have substituted $\tau=(2l/g)$ and $\nu=2\omega/g$. This integral can be again evaluated by completing the contour on the lower complex plane for $\nu>0$. (The function $\sinh(z-\mathrm{i}\varepsilon)$ vanishes along the imaginary axis for $y=-n\pi+\varepsilon$, with $n=0,1,\ldots,$ etc. Each of these is an eighth-order pole.) It can be shown (using Gradshteyn and Ryzhik 1965, p 305 3.314, p 950 8.384 (1), p 937 8.331, 8.332) that

$$I=\int_{-\infty}^{+\infty}\frac{\mathrm{e}^{-\mathrm{i}\rho x}\,\mathrm{d}x}{\sinh^{2n}(x-\mathrm{i}\varepsilon)}=\frac{(-1)^n}{(2n-1)!}\left(\frac{2\pi}{\rho}\right)\frac{1}{\mathrm{e}^{\pi\rho}-1}\prod_{k=1}^{n}[\rho^2+4(n-k)^2)] \tag{43}$$

so that R becomes

$$R_{\text{UAT}}(\omega)=\left(\frac{6\alpha^2}{35}\right)\frac{g^6}{\pi^3}\frac{\omega}{(\mathrm{e}^{2\pi\omega/g}-1)}\left(1+\frac{\omega^2}{g^2}\right)\left(1+\frac{\omega^2}{4g^2}\right)\left(1+\frac{\omega^2}{9g^2}\right). \tag{44}$$

(Note that R is positive definite and has the correct dimension.) *Equation (44) is a main result of this paper*; it shows that an accelerated detector coupled to the trace of $T^i{}_k$ will be excited in an accelerated frame. Moreover, the response function has a Planckian form with temperature $T=g/2\pi$ modified by a polynomial in ω.

Equipped with this knowledge, we can easily see that similar results should hold for the more general coupling $\mu^{ik}T_{ik}$ considered previously. In the general case we found that $R(\omega)$ is given by (equation (35))

$$R(\omega,\tau_1)=\frac{1}{2\pi^4}\int_{-\infty}^{+\infty}\mathrm{d}\tau\,\mathrm{e}^{-\mathrm{i}\omega\tau}\left(\frac{A-Bs^2+Cs^4}{s^{12}}\right). \tag{45}$$

The quantities A and B are quartic and quadratic expressions of the coordinates x^i and hence are analytic functions of τ. Therefore the poles of the integrand (in the lower complex plane) occur at the zeros of s^2, which are on the imaginary axis, at $z_n=-\mathrm{i}n\pi$. In other words $R(\omega)$ *will be non-zero* and has a Planckian form (due to C) modified by the A and B. This argument also shows that an ECD with any form of coupling will not click while on an inertial trajectory (because the pole of $(t-\mathrm{i}\varepsilon)^n$ is on the upper complex plane) which is rather reassuring.

Lastly, one may ask whether there exist other forms of μ^{i}_{j} for which $R(\omega)$ can be interpreted in a simple manner. We know that the only second-rank mixed tensor which has the same form in all frames should be proportional to δ^{i}_{j}. If we choose some other form for μ^{i}_{j} then the detector will have a simple interpretation *either* in the inertial frame *or* in the accelerated frame but *not* in both. Let us denote by $x^{i'}$ the Rindler coordinates, related to the inertial coordinates x^{i} by

$$x^0 = x^{1'} \sinh gx^{0'} \qquad x^1 = x^{1'} \cosh gx^{0'} \qquad x^2 = x^{2'} \qquad x^3 = x^{3'}. \tag{46}$$

If we assume, for example, that μ_{ij} has the form

$$\mu_{00} \neq 0 \qquad \mu_{ij} = 0 \qquad \text{for } i \neq 0, j \neq 0 \tag{47}$$

then one may consider the ECD to couple to the T^{00} component in the inertial frame. But in the accelerated frame $\mu_{i'j'}$ will have complicated components

$$(\mu'_{00}, \mu'_{11}, \mu'_{01}) \neq 0 \qquad \mu'_{ij} = 0 \qquad \text{for} \quad i, j \neq 0, 1 \tag{48}$$

so that $\mu_{i'j'} T^{i'j'} = \mu'_{00} T'^{00} + 2\mu'_{01} T'^{01} + \mu'_{11} T'^{11}$ couples to a strange mixture of T'^{ij} components. Of course, the combination $\mu_{ij} T^{ij}$ is covariant but it does not have the simple interpretation of measuring the same quantity in both the frames. This is in contrast to the case with $\mu^{i}_{j} \propto \delta^{i}_{j}$ where the ECD couples to the trace of T^{i}_{k} in all frames.

For the sake of illustration, we give below the resulting $R(\omega)$, when μ^{ij} has only the 00 component *in the accelerated frame*, i.e.

$$\mu'^{ij} = 0 \qquad \text{for } i', j' \neq 0 \qquad \mu'^{00} \neq 0 \tag{49}$$

so that it couples to the T'_{00} component *in the accelerated frame.* The μ^{ij} will have (μ^{00}, μ^{01} and μ^{11}) components non-zero in the inertial frame. (It is a very strange kind of detector in the inertial frame and the reader is justified in thinking that this is a mathematical artificiality. As we said before, we give this result purely as a curious illustration.) The $R(\omega)$ can be evaluated either by transforming μ^{ij} to the inertial frame and using our general result or by carefully transforming (18) to the Rindler frame and repeating the calculation. Either way, we obtain

$$R(\omega) = \left(\frac{\alpha^2}{336\pi^3}\right) \frac{g^6 \omega}{e^{2\pi\omega/g} - 1} \left(1 + \frac{\omega^2}{g^2}\right)\left(1 + \frac{3}{10}\frac{\omega^4}{g^4} - \frac{3}{10}\frac{\omega^2}{g^2}\right) \tag{50}$$

indicating a modified Planckian structure.

We shall now consider the consequences of our result.

4. Interpretation and discussion

In classical field theory simple relationships exist between a field, its energy density, momentum density, etc. If a classical field, say an electromagnetic field, vanishes then so does its energy density, momentum, etc. It is also usual to assume that the fields are observable by suitable arrangements. For example, a charged particle can be used to measure the electromagnetic field.

Quantum theory changes this situation with the introduction of the Fock basis and the concept of 'particles' associated with the field. Even though a state of 'no particles'—namely a vacuum—can be introduced, field fluctuations exist in this state. (Since the field ϕ and the number operator a^+a do not commute they cannot be simultaneously diagonalised.) The expectation value of the field does not determine the expectation value of other physical observables because of the existence of these fluctuations.

Quantum theory also changes the concept of measurements. A charged particle kept in an electromagnetic field interacts with the field via emission and absorption of photons. Therefore one may take the point of view that charged particles are actually detecting the photon content of a state rather than the field content of the state. *In other words, we lose all our ability to measure ϕ or $T^i{}_k(x)$ or any other observable directly*; any detector used to measure ϕ, $T^i{}_k(x)$, etc, can only couple such field variables via the exchange of quanta. This, however, does not create any special difficulty as long as we are only concerned with a Lorentz-invariant field theory. If particles are conceived as being Lorentz invariant, then all inertial observers will agree on the results of the measurement of any observable.

The trouble begins when we introduce accelerated observers. Since the concept of a particle is not generally covariant, accelerated observers will see a different particle content compared to an inertial observer in any quantum state. What is more, the accelerated and inertial observers will arrive at *different* results for the measurement of *any* physical observable. This is a direct consequence of the philosophy that detectors interact with observables only via the emission and absorption of quanta. In other words, we have a complete breakdown of operational general covariance.

It is convenient (and necessary) to distinguish between 'formal covariance' and 'operational covariance' of an observable. The number operator (a^+a) for example is *neither* formally *nor* operationally covariant. (Every observer defines this observable differently.) On the other hand, the energy-momentum tensor T_{ik} or some scalar functional $A(\phi)$ of the field ϕ are *formally* covariant objects. These operators as well as their expectation values $\langle\psi|T_{ik}|\psi\rangle$ transform in a systematic tensorial manner. It has always been assumed that if T_{ik} and T'_{ik} are obtained from one another by a tensorial coordinate transformation then the observers using the corresponding coordinates x^i and x'^i *will actually measure* the values as T_{ik} and T'_{ik}. This assumption (valid classically) is not valid in quantum theory. Any operational procedure devised to measure T_{ik}—say, by a detector coupled to T_{ik} like the ECD—will go about performing this measurement by emission and absorption of quanta and hence will respond differently in inertial and accelerated trajectories. In other words, even though the *formal* covariance is assured by the relation

$$\langle 0|T_{ik}|0\rangle_{\text{reg}} = 0 = \langle 0|T'_{ik}|0\rangle_{\text{reg}} \tag{51}$$

the operational covariance is completely lost in quantum theory. The objects like $\langle 0|T'_{ik}|0\rangle$ have no operational significance.

Any reader who suspects the above conclusion is strongly urged to construct a detector model which will satisfy the following criteria: (i) it detects T_{ik} of a field in the inertial frame (and hence does not click in the inertial vacuum), (ii) it does *not* click when accelerated and (iii) the coupling is local, causal and does not involve expectation values of operators. (For example, an unusual detector coupling like $\mu_{ik}\langle 0|T^{ik}|0\rangle_{\text{reg}}$ will remain zero in all frames, but it is not a realisable operator-operator coupling.) We do not believe it is possible to construct a detector satisfying these criteria. In other words, formal expressions like $\langle T_{ik}\rangle_{\text{reg}}$ have no operational significance.

Does this mean that quantum theory cannot be covariant at all? That is not necessarily the case. Most of the comments in the previous paragraph apply to situations in which the quantum state of the system is in a Fock basis state. Fock basis is introduced as the eigenbasis of the number operator, which—we know—is not covariant. The situation can be improved using the basis in which the field operator

$\phi(x)$ is diagonal. The response of detectors in such states form an interesting subject of study; the results will be published separately.

One might question whether the ECD discussed in this paper should be considered as 'measuring' T_{ik}. For example, one can invoke the notion that 'something measures T_{ik} only if it gives T_{ik} as the answer'. We do not think this attitude helps matters in any way. (i) Firstly, it would be impossible to construct a detector which measures *only* T_{ik} in every frame. One has, therefore, to use (presumably) different forms of coupling for detectors in different trajectories. (ii) Secondly, note that a detector like ECD does measure T_{ik} in the classical or semiclassical limit. The failure of the ECD to respond to T_{ik} *alone* is a feature directly related to the inevitability of quantum fluctuations. We believe that this feature has some fundamental significance.

Finally, we would like to comment on the role of gravity and the expectation value of T_{ik}. One may ask: can one observe the value of $\langle 0|T_{ik}|0\rangle_{\text{reg}}$ by measuring the gravitational field it produces? In fact, one can, but not in a fundamental manner. This can easily be seen by considering the linearised limit of Einstein's equations

$$\Box \bar{h}_{ik} = \langle 0|T_{ik}|0\rangle_{\text{reg}}. \tag{52}$$

This is equivalent to a detector coupling (remember that in this case our 'detector' is the gravitational field $\bar{h}_{ik}$) of the form $\bar{h}_{ik}\langle 0|T^{ik}|0\rangle_{\text{reg}}$. This is purely classical and cannot be fundamental. On the other hand, one may try to work out the gravitational fluctuations in the weak field limit by using equation (52) without the expectation value on the right-hand side. Then one would use a fully quantum mechanical coupling and the existence of non-zero correlation $\langle 0|h_{ik}h_{lm}|0\rangle$ would signal the existence of fluctuations in T_{ik}. Thus, even though a semiclassical gravitational field h_{ik} of (52) might give some information about T_{ik}, it does *not* provide the kind of information which we expect from a detector measuring T_{ik}. These questions, as well as the behaviour of the ECD in curved spacetime, are under investigation.

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