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Lifetime of a black hole

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We study the constraints placed by quantum mechanics upon the lifetime of a black hole. In the context of a moving-mirror analog model for the Hawking radiation process, we conclude that the period of Hawking radiation must be followed by a much longer period during which the remnant mass (of order m_P) may be radiated away. We are able to place a lower bound on the time required for this radiation process, which translates into a lower bound for the lifetime of the black hole. Particles which are emitted during the decay of the remnant, like the particles which comprise the Hawking flux, may be uncorrelated with each other. But each particle emitted from the decaying remnant is correlated with one particle emitted as Hawking radiation. The state which results after the remnant has evaporated is one which locally appears to be thermal, but which on a much larger scale is marked by extensive correlations.

I. INTRODUCTION

The theories of gravitation and of quantum mechanics form two of the greatest achievements of twentieth century physics. Although a field-theory generalization of quantum mechanics can describe the strong, weak, and electromagnetic interactions, there is as yet no fully developed quantum theory of gravity. Recent developments in the theory of strings[1] offer candidates for such a theory, and it is certainly timely to consider the paradoxes which arise when one considers the confrontation between quantum mechanics and gravity.

One such paradox was pointed out by Hawking[2] in the course of his seminal work on black-hole radiation. Hawking examined the evolution of a quantum field in the background metric of a classical black hole and discovered that particle creation should take place. The emitted particles were found[3] to build a thermal spectrum and to be uncorrelated among themselves. As the hole loses mass through this mechanism, its temperature rises and Hawking speculated[4] that this would lead to the explosive disappearance of the black hole shortly after its mass reached a value on the order of the Planck mass m_P. If this were to happen, there would be left behind a residue of thermal radiation, which is a mixed state in quantum-mechanical terms. Since the original black hole might have formed from a dust cloud described as a quantum-mechanical pure state, the black-hole explosion could convert a pure state into a mixed state. But such a conversion would violate the fundamental tenets of quantum mechanics, and this led Hawking to suggest that these tenets must be revised.

We will adopt a less radical view in this paper in assuming that a quantum-mechanical description is always possible and asking what aspects of Hawking's scenario are thereby violated. We find that the black-hole remnant of mass m_P cannot disappear explosively but must slowly dissipate its remaining energy. Appealing to a mechanical analog of the Hawking process, we are able to place a lower bound on the lifetime of the black-hole remnant. And assuming that this bound is saturated, we can fully characterize the radiation which remains after the remnant has evaporated.

The state which results from our model of black-hole evaporation is one which is locally thermal. Radiation emitted during the Hawking phase is locally thermal with a temperature which increases as the mass of the emitting black hole decreases. Radiation emitted from the decaying remnant can also be locally thermal but at a much lower temperature. There are no local correlations among the emitted particles, but each particle emitted during the Hawking phase is correlated with some particle emitted from the decaying remnant. These long-range correlations are a primary characteristic of the state which results from the complete evaporation of a black hole. The state is an unusual one, but still a legitimate quantum-mechanical state. The time required to form this state is large, and the length of time required for the black-hole remnant to decay is always much larger than the time required for the original black hole to decay (via Hawking radiation[5]) and form the remnant. Thus one can conclude that the period of Hawking radiation is a relatively brief period in the life of a quantum black hole.

The mechanical model upon which we rely for an intuitive grasp of the Hawking process is one which has been studied extensively in the literature.[6] It involves a quantum field formulated upon a two-dimensional flat spacetime which is bounded by a moving, reflecting wall. In the preceding paper[7] we have reviewed the salient features of this model and argued its suitability as an analog to the Hawking process. In the following section we collect and generalize the results of our preceding paper. In Sec. III we pose, in the context of our model, the question of what must happen if a black hole radiates away all its mass. We answer the question by establishing a lower bound on the lifetime of the black hole and by making a simple characterization of the final state which results after the hole has disappeared. We argue in Sec. IV that our arguments may be applied with equal force to realistic models of gravitational collapse in four-dimensional spacetimes. A final section contains some general remarks on the implications of our results for the structure of quantum theories of gravity.

II. MOVING MIRRORS

The phenomenon of particle production in the changing gravitational field of a collapsing massive object can be mimicked by a simple model[6] in two-dimensional flat spacetime. To this end we consider a free quantum field constrained by a movable, reflecting boundary. For simplicity we assume that the quantum field describes massless scalar particles. The trajectory of the moving mirror can be described in terms of null cordinates u and v by the equation

$$v=p(u)\ . \tag{2.1}$$

If the mirror undergoes an acceleration, field quanta will be produced a the mirror surface, producing an energy flux

$$\langle T_{uu}\rangle=\frac{1}{12\pi}(p')^{1/2}\partial_u{}^2(p')^{-1/2}\ . \tag{2.2}$$

By an appropriate choice[6] of the function $p(u)$, this flux can be made to match the radial flux from a collapsing mass, as predicted by the arguments of Hawking.[2]

In either Hawking's model or the moving-mirror model, particle production can be explained in terms of a Bogoliubov transformation relating zero-particle states defined on null surfaces $\mathscr{I}^-$ and $\mathscr{I}^+$ which lie, respectively, in the distant past or distant future. The zero-particle state on $\mathscr{I}^-$ is naturally interpreted as the vacuum state of the field ϕ. Viewed from $\mathscr{I}^+$, this state has the form

$$|\,\text{vac}\rangle=U\,|\,00\rangle\ . \tag{2.3}$$

Here $|\,00\rangle$ denotes the zero-particle state on $\mathscr{I}^+=\mathscr{I}_R^+\cup\mathscr{I}_L^+$, and U is the Bogoliubov transformation, which populates $|\,00\rangle$ with correlated pairs of right- and left-moving particles.

If the trajectory function $p(u)$ has the simple form

$$p(u)=-\kappa^{-1}e^{-\kappa u}\ , \tag{2.4}$$

then the Bogoliubov transformation can be diagonalized, and the correlations on $\mathscr{I}^+$ can be exhibited in explicit detail. This procedure has been carried out in the previous paper.[7] The physical picture which was developed in that paper will be of considerable use in what follows. Consider a wave packet which leaves the mirror in the vicinity of the ray u. The outgoing wave packet can be traced back to a packet of identical shape in terms of a variable

$$V=-\kappa^{-1}\ln(-\kappa v)\ , \tag{2.5}$$

which corresponds to the inverse of Eqs. (2.1) and (2.4). The singularity in V at $v=0$ corresponds to the fact that the trajectory (2.4) has a horizon, in the sense that rays with $v>0$ miss the mirror entirely.

The sharp boundary at $v=0$ is not without consequence for the wave packet discussed above. If we perform an analytic continuation in v, we can examine the form of the packet in the region $v>0$. It does not vanish, but exhibits the same form in terms of a variable

$$W=\kappa^{-1}\ln(\kappa v)\ , \tag{2.6}$$

as it did in u or V. The amplitude of the $v>0$ component relative to the $v<0$ component is $e^{\pm\pi\omega/\kappa}$, with a sign in the exponent which depends upon the sense of the continuation around the singularity at $v=0$. This sign can be related to the relative signs of the energies of the wave packets on $\mathscr{I}^+$ and $\mathscr{I}^-$, as we have described in our previous work. Here we will simply note that the variables V and W provide a convenient means for diagonalizing the Bogoliubov transformation. The state [Eq. (2.3)] which results on $\mathscr{I}^+=\mathscr{I}_R^+\cup\mathscr{I}_L^+$ contains correlated left- and right-moving quanta. Specifically, a right-moving packet constructed in terms of the variable u is paired with an identical left-moving packet constructed in terms of the variable W.

The width of the correlation (in u or W) is of order κ^{-1}. Furthermore, the dominant frequencies of quanta on $\mathscr{I}_R^+$ are of order κ. These observations allow us to generalize from the trajectory (2.4) toward a more realistic analog to the Hawking process itself. The trajectory (2.4) corresponds to a constant Hawking temperature

$$T_H=\frac{\kappa}{2\pi}\ . \tag{2.7}$$

Since the Hawking temperature actually depends upon the mass of the collapsing object

$$T_H=\frac{m_P{}^2}{8\pi M}\ , \tag{2.8}$$

this temperature should increase as mass is gradually lost to Hawking radiation. In terms of our moving-mirror model, we have a local acceleration

$$\kappa(u)=-\partial_u\ln p'(u)\ , \tag{2.9}$$

which induces correlations over a region in u of width κ^{-1}. Since the dominant wavelengths of the emitted quanta are also of order κ^{-1}, it follows that our qualitative arguments about the structure of the state on $\mathscr{I}^+$ are unaffected as long as

$$\partial_u\kappa(u)\ll\kappa^2(u)\ . \tag{2.10}$$

This inequality demands simply that the local temperature should not vary appreciably over the de Broglie wavelength of the emitted quanta. Let us now allow M to be a function of u and identify the rate of mass loss with the energy flux (2.2):

$$\partial_u M(u) = -\frac{\kappa^2(u)}{48\pi} . \quad (2.11)$$

In writing this equation we have used Eq. (2.9) and have dropped a total divergence from Eq. (2.2) to render the energy flux explicitly positive. If now we combine Eqs. (2.7), (2.8), and (2.11), we can deduce the explicit form of $M(u)$:

$$M^3(u) = M_0{}^3 - \frac{m_P{}^4 u}{256\pi} . \quad (2.12)$$

This form is valid only as long as

$$M(u) \gg m_P , \quad (2.13)$$

since when $M(u)$ approaches m_P, quantum gravitational effects are likely to invalidate the semiclassical expression (2.8). Given our identification

$$\kappa(u) = m_P{}^2/4M(u) , \quad (2.14)$$

we see that the inequality (2.10) follows from the presumed inequality (2.13).

Equations (2.14) and (2.12) specify the local acceleration $\kappa(u)$ of the mirror which correctly mimics the particle production of the Hawking process. Introducing this acceleration in Eq. (2.9), we can compute the mirror trajectory in the analog model for the period

$$0 \le u \le u_1 \ll 256\pi M_0{}^3/m_P{}^4 . \quad (2.15)$$

The last inequality is introduced to enforce the condition (2.13). The differential equation for $p'(u)$, Eq. (2.9), can be integrated to obtain

$$p'(u) = \exp\left[\frac{96\pi}{m_P{}^2}[M^2(u) - M_0{}^2]\right] . \quad (2.16)$$

Subject to the condition (2.13), this can be integrated again to give

$$p(u) \simeq -\frac{4M(u)}{m_P{}^2} \exp\left[\frac{96\pi}{m_P{}^2}[M^2(u) - M_0{}^2]\right] . \quad (2.17)$$

Note that using Eqs. (2.14), (2.16), and (2.17), we can construct the ratio

$$\frac{p'(u)}{p(u)} = -\kappa(u) . \quad (2.18)$$

This ratio has the same functional form as for a trajectory (2.4) with constant acceleration, a fact which we will exploit in the following section. Note further that in any region of u of width $\kappa^{-1}(u^*)$ about some central value u^*, one can approximate

$$p(u) \simeq -\kappa^{-1}(u^*) e^{-\kappa(u^*)(u-u^*)} p'(u^*) . \quad (2.19)$$

This implies that at any position along the trajectory $p(u)$, the apparent horizon—or apparent asymptote of $p(u)$—is at $v=0$. This again is the same as for the trajectory (2.4). It implies that the variables V and W of Eqs. (2.5) and (2.6) remain relevant for diagonalizing the Bogoliubov transformation of the moving mirror, even when the acceleration is nonuniform.

The physical picture which results for the spacetime slice $0 \le u \le u_1$ is as follows. In the vicinity of a ray labeled by the null coordinate u, there are particles streaming out from the mirror which have energies of order $\kappa(u)$. These particles have no local correlations and appear locally to originate from a thermal source of temperature $\kappa(u)/2\pi$. These particles are, however, correlated with left-moving quanta. The dominant correlations[7] involve quanta in the vicinity of a ray

$$v = -p(u) , \quad (2.20)$$

which have frequencies of order

$$\kappa(u)/p'(u) . \quad (2.21)$$

This follows from the fact[7] that a given right-moving wave packet is correlated with a left-moving packet which is narrower by a factor $p'(u)$. In the next section we will investigate the field configuration for $u > u_1$ and investigate the fate of the virtual quanta described by Eqs. (2.20) and (2.21).

III. LATE-TIME BEHAVIOR

For $u \gg u_1$, Hawking's formula (2.8) no longer applies and we can no longer determine the explicit form of the trajectory $p(u)$. We can, however, deduce the asymptotic form that $p(u)$ must possess and join the asymptotic trajectory on to the trajectory (2.17) subject to the constraint of energy conservation in the system modeled by the moving mirror. This constraint is strong enough to provide a lower bound on the lifetime of the remnant found at time u_1.

Let us assume that the remnant can radiate away its entire mass in some finite-time interval to leave behind a flat region of spacetime. In our analog model this corresponds to a mirror which is stationary. Indeed, $p''(u)$ must vanish so that particle production can cease and $p'(u)$ must equal unity so that rays passing through the region once occupied by the hole no longer suffer any red-shift. At $u = u_1$, the red-shift factor $1/p'(u_1)$ is very large, so there must be a period of deceleration which follows u_1 and allows $p'(u)$ to approach its asymptotic value. This deceleration cannot be very great, however, since the amount of energy radiated in the course of this deceleration must be equal to the available mass $M(u_1) = M_1$. The trajectory which maximizes the allowed deceleration can be found with the aid of Eq. (2.2). Demanding continuity of $p(u)$ and $p'(u)$ at $u = u_1$, one obtains a trajectory

$$p(u) = \kappa_2{}^{-1} e^{\kappa_2(u-u_1) - \chi_1} - (\kappa_2{}^{-1} + \kappa_1{}^{-1}) e^{-\chi_1} , \quad (3.1)$$

with a constant deceleration κ_2 and parameters

$$\kappa_1 = \kappa(u_1) \quad (3.2)$$

and

$$\chi_1 = -\ln[p'(u_1)] \simeq 96\pi(M_0{}^2 - M_1{}^2)/m_P{}^2 . \quad (3.3)$$

We will ignore the effects of the discontinuity in $p''(u)$ which results from joining the trajectories (2.17) and

(3.1) at $u = u_1$. Smoothing the trajectory to eliminate this kink would affect only those few quanta emitted very close to u_1.

The trajectory (3.1) attains the final value $p' = 1$ at

$$u_2 = u_1 + \chi_1/\kappa_2 \ . \tag{3.4}$$

During the period $u_1 \le u \le u_2$ the mirror trajectory (3.1) produces a constant flux of particles with a total energy

$$\int_{u_1}^{u_2} du\, T_{uu} = \frac{(u_2 - u_1)\kappa_2^2}{48\pi} = \frac{\chi_1 \kappa_2}{48\pi} \ . \tag{3.5}$$

This quantity should equal the mass at $u = u_1$, whence

$$\kappa_2 = \frac{48\pi M_1}{\chi_1} \simeq \frac{M_1}{2M_0}\frac{m_P^2}{M_0} \ . \tag{3.6}$$

Note that this acceleration is much smaller than the Hawking acceleration at $u = 0$:

$$\kappa_0 = \kappa(0) = \frac{m_P^2}{4M_0} \ . \tag{3.7}$$

Therefore, the temperature at which the remnant evaporates is much smaller than the temperature of the Hawking radiation. Consequently the time interval

$$u_2 - u_1 = \frac{\chi_1}{\kappa_2} \simeq \frac{3}{4}\frac{m_0}{M_1} u_1 \ , \tag{3.8}$$

during which the remnant mass M_1 is released, is much longer than the interval u_1 during which the remnant formed.

We would like next to display the structure of the state which results after the mirror has come to rest. To do this we need only trace the path of the virtual left-moving quanta discussed in the previous section. There we argued that, correlated with the real quanta emitted at some $u < u_1$, there are virtual quanta moving near the ray $v = -p(u)$ with energies of order $\kappa(u)/p'(u)$. These quanta will strike the mirror during the decelerating portion of its trajectory at a time $u^* > u_1$ such that

$$p(u^*) = -p(u) \ . \tag{3.9}$$

We assume that

$$\kappa_2(u^* - u_1) \gg 1 \ , \tag{3.10}$$

so that we can neglect the constant term in Eq. (3.1). This allows us to write the red-shift factor at u^* in the form

$$p'(u^*) = -\kappa_2 p(u) = \frac{\kappa_2}{\kappa(u)} p'(u) \ , \tag{3.11}$$

where we have invoked Eq. (2.18) to relate $p(u)$ and $p'(u)$. It follows from Eq. (3.11) that, following their reflection from the mirror, the virtual quanta should attain energies of order κ_2. These quanta correspond to the real quanta emitted in consequence of the mirror's deceleration. Quanta emitted in the period $u_1 \le u \le u_2$ are locally thermal, with a temperature

$$T_2 = \frac{\kappa_2}{2\pi} \ . \tag{3.12}$$

But each of the quanta emitted in this period is correlated with some quantum in the Hawking flux. Particles emitted in the vicinity of u^* are correlated with particles at u, where u^* and u are related by Eq. (3.9). The width of the correlations extends over distances of order κ_2^{-1} about u^* and $\kappa^{-1}(u)$ about u, as discussed in our previous work.[7] Note that u^* and u are typically very widely separated. Therefore, the final state of our moving-mirror system is characterized by correlations over very large distances, as illustrated in Fig. 1.

These correlations permit the final state of the scalar field to be, just like the initial state, a pure quantum state. Locally the state appears to be thermal and hence a mixed quantum state. But although there are no local correlations, there are, as we have seen, extensive long-range correlations. This is an unusual state. More typically one expects major short-range correlations and vanishing long-range correlations. Here the situation is reversed. We conclude that, within the context of the moving-mirror model, evaporation of a black hole does not induce the evolution of a pure quantum state into a mixed state. Rather, it produces a pure state with unusual long-range correlations. In the following section we will argue that similar conclusions may be reached in the realistic case of a collapsing massive object. There the correlations may be viewed as a consequence of the black hole's ability to induce large red-shifts and large time dilations for particles which pass close by it.

IV. GENERALIZATIONS

In the previous section we have deduced the structure of the state which results in the moving-mirror analog of the total evaporation of a black hole. Here we would like to generalize these results so that they might be applied to the realistic problem of a collapsing massive object. To do so, we first note some general features of the picture developed in the previous section. We had a

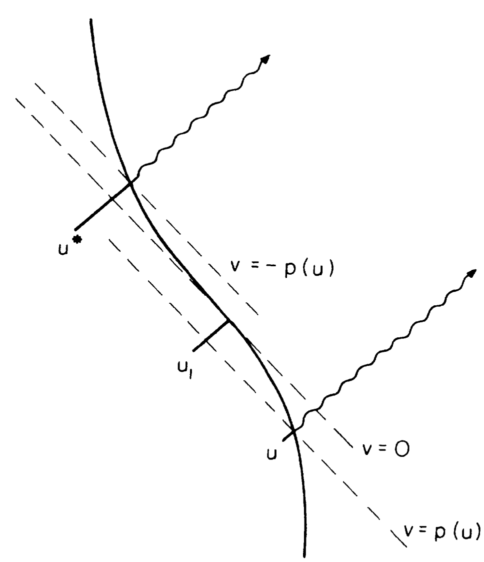

FIG. 1. Trajectory of the moving mirror. Quanta emitted at u are correlated with quanta of a much lower energy which are emitted at u^*.

remnant of mass M_1, which decayed in a time u_2-u_1 with the emission of

$$N \sim M_1/\kappa_2 \tag{4.1}$$

uncorrelated quanta. An emitted particle has a typical energy of order κ_2 and hence can be formed on a time scale no shorter than $\kappa_2{}^{-1}$. The minimum time required for the emission of N uncorrelated quanta is thus

$$N/\kappa_2 \sim M_1/\kappa_2{}^2 \ . \tag{4.2}$$

We can compare this with

$$u_2-u_1=\chi_1/\kappa_2 \sim \frac{M_1}{\kappa_2{}^2} \ . \tag{4.3}$$

Therefore, the origin of a lower bound on the lifetime of the remnant can be traced to the need for emitting N uncorrelated low-energy quanta. Each of these quanta is correlated with some quantum in the Hawking flux, so we can regard N equally well as the number of quanta emitted in forming the remnant. Indeed, from Eqs. (4.1) and (3.6) and Eqs. (3.3) and (2.16) we have

$$N \sim \chi_1 \sim \frac{M_0{}^2-M_1{}^2}{m_P{}^2} \ . \tag{4.4}$$

During the interval $0 \leq u \leq u_1$ the particle flux has the form

$$\frac{dN}{du} \sim -\kappa^{-1}(u)\frac{dM}{du} \simeq -\frac{2}{m_P{}^2}\frac{dM^2}{du} \ . \tag{4.5}$$

Upon integration, this expression reproduces Eq. (4.4).

We can now use the moving-mirror model to describe the decay of a black hole of mass $M_0 \gg m_P$. The spatial coordinate of the analog system assumes the role of a radial coordinate for the metric which describes the collapsing system. Through the Hawking process the hole radiates (locally) thermal quanta to reduce its mass to a value M_1, with $m_P \ll M_1 \ll M_0$. The real quanta radiated in the Hawking process are correlated with virtual quanta falling toward an apparent horizon. When the remnant M_1 has formed, this horizon will lie at a radius

$$R_1=\frac{2M_1}{m_P{}^2} \ . \tag{4.6}$$

If the horizon is to disappear, the virtual quanta must be radiated as real quanta with a total energy equal to M_1. The typical energy of these real quanta is

$$\frac{M_1}{N} \sim \kappa_2 \sim \frac{M_1 m_P{}^2}{M_0{}^2} \ . \tag{4.7}$$

The corresponding wavelength is larger, by a factor of order $(M_0/M_1)^2$, than the radius R_1. It follows that emission should proceed predominantly from states of zero angular momentum. We can compute the minimum time for the emission of N uncorrelated quanta and reproduce the result of our one-dimensional problem, Eq. (4.3). Note that this approach allows us to bypass an explicit construction of the late-time modes of the system. Thus we can avoid having to deal directly with the state of the gravitational field in the region where classical gravity is inapplicable. Rather, we approach it indirectly, constraining the system to conserve its total energy and observing it, in effect, through the structure of the Bogoliubov transformation which it induces.

V. DISCUSSION

In the preceding sections we have described how a black hole might decay in a manner consistent with the laws of quantum mechanics. Particle production in our model is associated with the existence of an apparent horizon. The particles are produced in pairs: one particle is emitted directly and the other approaches the horizon and is emitted after a lengthy time delay at a considerably reduced frequency. No real horizon exists in our model. Note that our description of the scalar field, from which these particles arise, is fully quantum mechanical. Hence one may regard our results as constraints placed on the final evolution of a black hole by the laws of quantum mechanics. If the remnant of mass M_1 were to disappear in a time of order $1/M_1$, as envisioned by Hawking, then these quantum laws would have to be violated. What we have shown is that there is another possibility—that the remnant might slowly decay (in a time of order u_2-u_1) and that the laws of quantum mechanics should remain intact. Which of these pictures is correct will be determined only by the construction of a full theory which merges, insofar as possible, the theories of quantum mechanics and gravity.

This paper does point out some aspects to be looked for in such a theory of quantum gravity. If our model is correct, then at the Planck scale gravitational forces must exist which reverse the accelerating collapse indicated by Einstein's classical theory (for $M \gg m_P$) and which induce a Bogoliubov transformation of the type discussed in Sec. III. Note that the temperature at which the remnant M_1 decays is a function of the initial mass M_0 and is thus, in principle, an arbitrarily small quantity. It follows that any fields relevant for the decay of the remnant must have strictly massless quanta. If the collapsing object is comprised of ordinary matter, then it will possess large initial values of baryon number B and of the difference of baryon and lepton numbers $B-L$. Since there are no massless baryons, it follows that baryon number cannot be conserved in the process of gravitational collapse and subsequent black-hole evaporation. Furthermore, if a grand unified theory which incorporates quantum gravity is to conserve the difference of baryon and lepton numbers, then there must be strictly massless leptons in the theory. This is a constraint not usually given by models of grand unification.

Suppose now that we were to adopt Hawking's viewpoint and assume that the usual laws of quantum evolution are invalid. This paper highlights the features that such a theory of gravity would have to possess. Specifically, there must be some physical mechanism which destroys the correlation between real radiated Hawking quanta and virtual quanta falling toward the horizon. Such a mechanism could be provided by describing the black hole itself as a thermal bath of gravitons. Then truly thermal radiation of various coupled fields could occur in the same manner as that in which other physical systems approach their equilibrium state. In this fashion one could associate an entropy with the quantum black hole, which would be transferred to the coupled quantum fields as the black hole decayed. Note that in the model developed here this is not what happens. There is no entropy associated with the emitted particles of the quantum field, which remains in the same pure state. Therefore, it is inappropriate in our approach to associate an entropy with the quantum black hole. An observer restricted to a limited spacetime region could not observe the correlations which characterize the final state into which the black hole decays and might assign a nonzero entropy to the apparently thermal radiation that he detects. But this is the usual entropy of statistical mechanics which is associated[8] with the incomplete information available to the observer; it is not an intrinsic property of the quantum state of the black hole or of the radiation which it emits.

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[5]We will use the phrase "Hawking radiation" to describe quanta emitted in response to the acceleration of a semiclassical gravitational field. Hence we confine our use of this phrase to particle emission from objects with masses large relative to the Planck mass m_P.

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